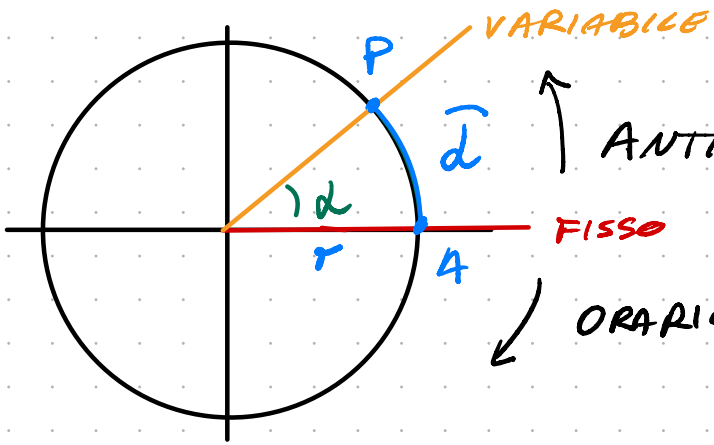
LEZIONE 5 – FUNZIONI TRIGONOMETRICHE

CIRCONFERENZA GONIOMETRICA → $360° = 2\pi$



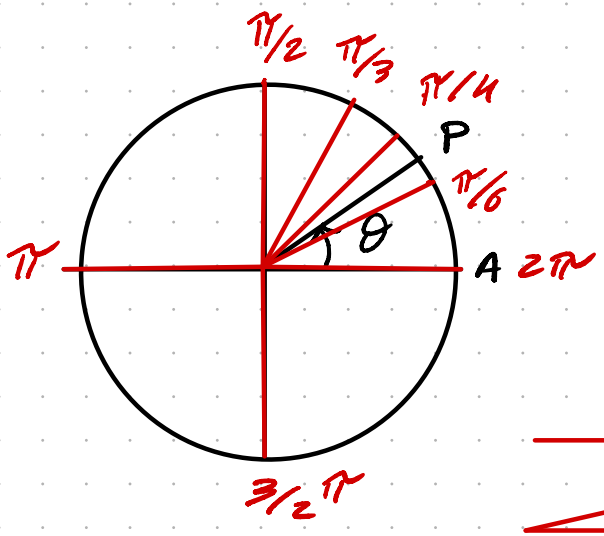
$$\text{RADIANTI} = \frac{\overset{\frown}{\alpha}}{r}$$

$\overset{\frown}{\alpha}$ → lunghezza dell'arco $\overset{\frown}{AP}$

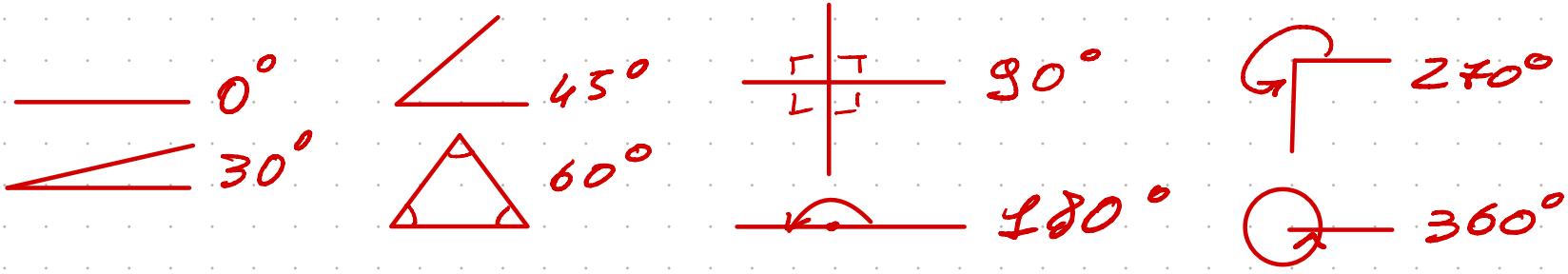
r → raggio

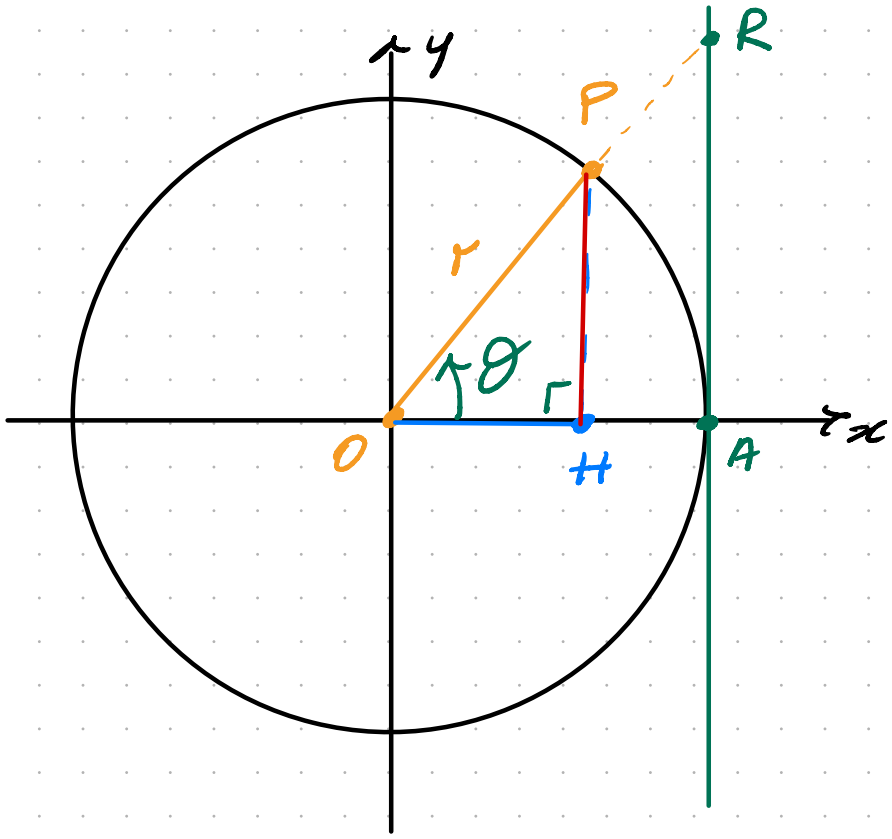
ex

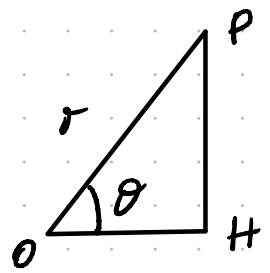
$\alpha = 45° \rightarrow \ell_{\overset{\frown}{AP}} = \dfrac{\frac{1}{8}\, 2\pi r}{r} = \dfrac{\pi}{4}$



Misure								
GRADI $\theta(°)$	0	30	45	60	90	180	270	360
RADIANTI θ (rad)	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	π	$\frac{3}{2}\pi$	2π







$\overline{PH} = \overline{OP}\sin\theta \rightarrow \quad \text{Sen}\,\theta = \frac{\overline{PH}}{r}$ $\quad (\overline{OP} = r)$

$\overline{OH} = \overline{OP}\cos\theta \rightarrow \quad \cos\theta = \frac{\overline{OH}}{r}$

$\overline{PH} = \overline{OH}\tan\theta$

$\hookrightarrow \tan\theta = \frac{\overline{PH}}{\overline{OH}} = \frac{r\sin\theta}{r\cos\theta} = \frac{\overline{RA}}{r}$

CIRCONFERENZA UNITARIA ($r=1$)

$\sin\theta = \overline{PH}$

$\cos\theta = \overline{OH}$

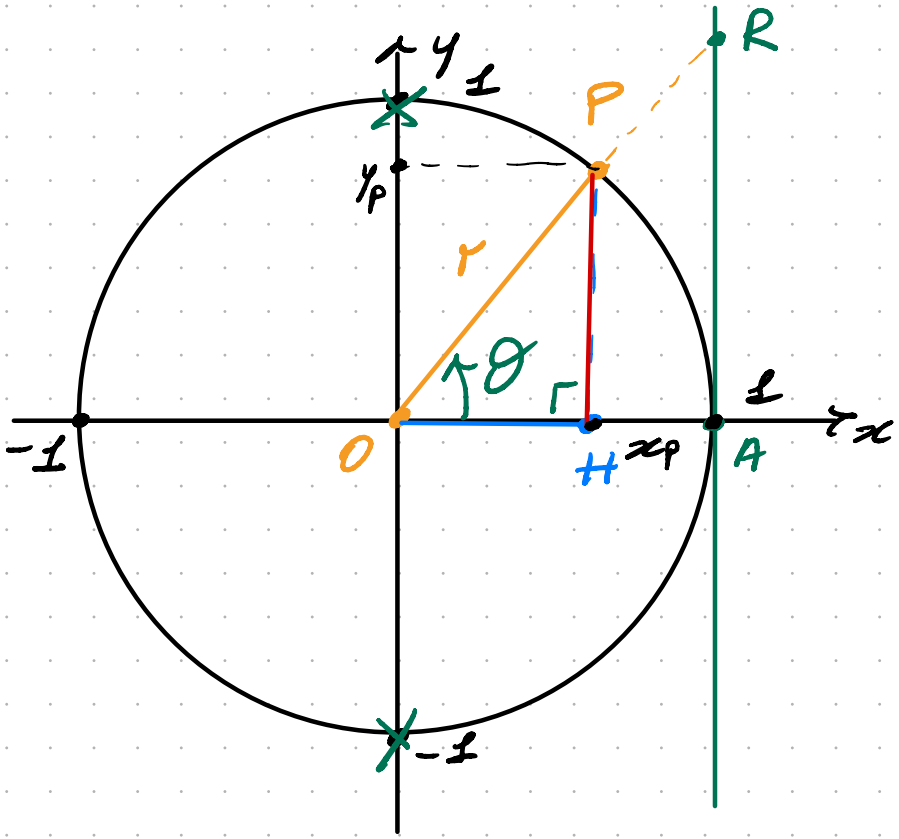
$\tan\theta = \overline{RA}$

$\rightarrow$

Cosecante $\quad \text{cosec}\,\theta := \frac{1}{\sin\theta}$

Secante $\quad \sec\theta := \frac{1}{\cos\theta}$

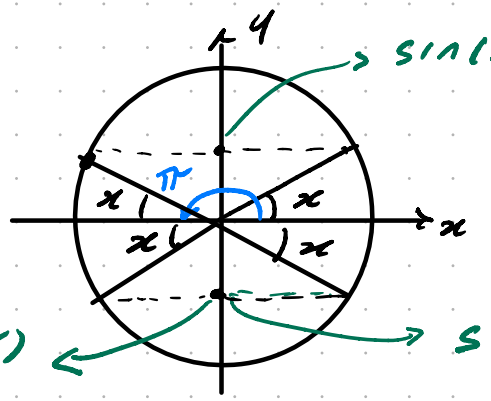
Cotangente $\quad \cot\theta := \frac{1}{\tan\theta}$



- $-1 \leq \sin\theta \leq 1$
- $-1 \leq \cos\theta \leq 1$
- $\ln \theta = \pi/2$ e $\theta = 3/2\pi$ $\nexists$ $\tan\theta$

$x^2 + y^2 = 1 \rightarrow \sin^2\theta + \cos^2\theta = 1$

- $\sin(x+\pi) = -\sin(x)$



$\sin(x) = \sin(\pi - x)$

=> VERA

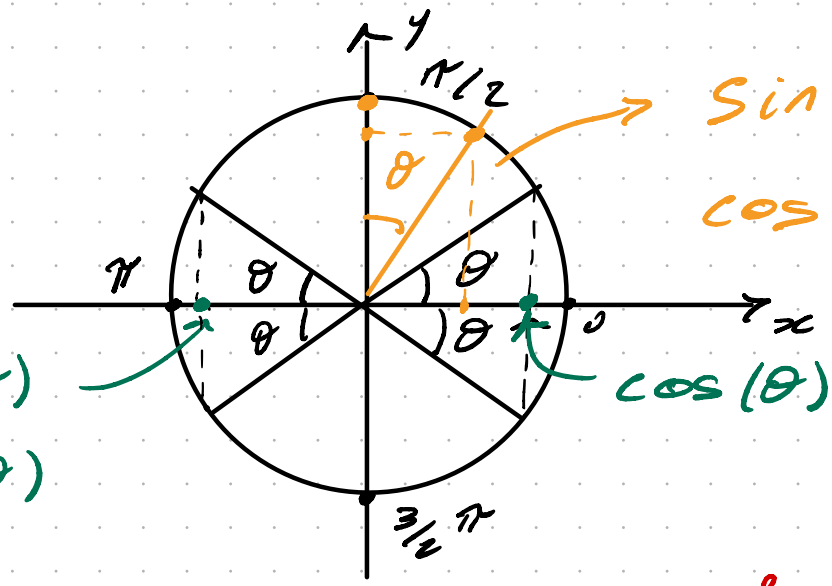
$-\sin(x) = \sin(x+\pi)$

$\sin(2\pi - x) = \sin(-x)$

$= -\sin(x)$

↳ ex $\sin(\pi - x) = \sin(x)$?

- $\cos(\pi-\theta) = \cos\theta$? NO → $\cos(\pi-\theta) = -\cos\theta$
- $\cos(\pi+\theta) = -\cos\theta$? SI !!!



$\sin\left(\frac{\pi}{2}-\theta\right) = \cos(\theta)$

$\cos\left(\frac{\pi}{2}-\theta\right) = \sin(\theta)$

$-\cos(\theta) = \cos(\pi+\theta)$

$\overset{!}{=} \cos(\pi-\theta)$

$\cos(\theta) = \cos(2\pi-\theta)$

$\overset{!}{=} \cos(-\theta)$

f è dispari se $f(-x) = -f(x)$

f è pari se $f(-x) = f(x)$

RICAPITOLANDO

$\sin(\pi-\theta) = \sin\theta$

$\cos(\pi-\theta) = -\cos\theta$

$\sin(-\theta) = -\sin(\theta)$ ⇒ DISPARI

$\cos(-\theta) = \cos\theta$ ⇒ PARI

$\sin(\pi+\theta) = -\sin\theta$

$\cos(\pi+\theta) = -\cos\theta$

$\sin\left(\pi/2-\theta\right) = \cos\theta$

$\cos\left(\pi/2-\theta\right) = \sin\theta$

FORMULE DI (+) E (−)

$$\sin(x \pm y) = \sin x \cdot \cos y \pm \cos x \sin y$$

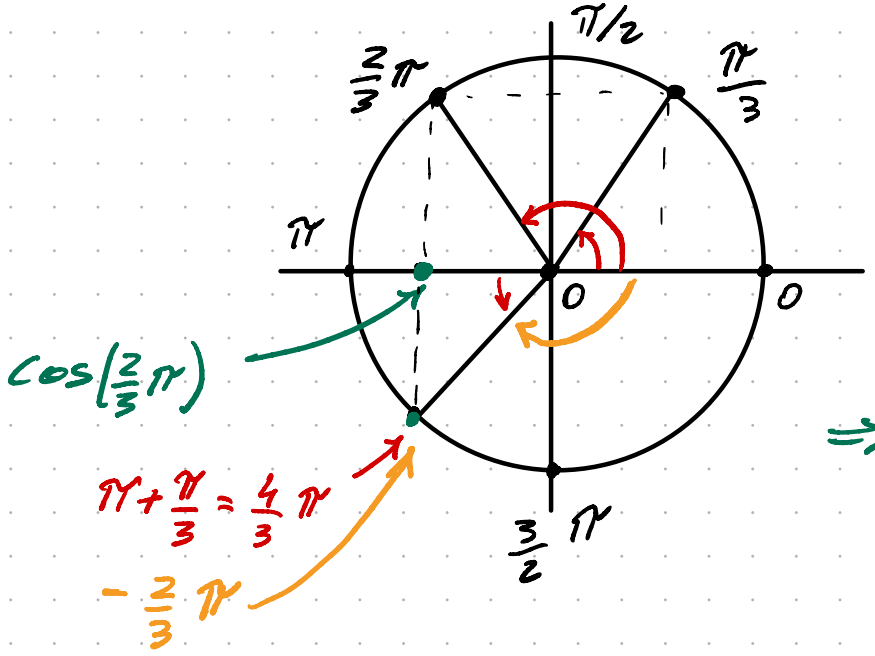
$$\cos(x \pm y) = \cos x \cdot \cos y \mp \sin x \sin y$$

- $\cos(2\alpha) =$

$$= \cos(\alpha + \alpha) = \cos\alpha \cdot \cos\alpha - \sin\alpha \cdot \sin\alpha$$

$$\overset{!}{=} \cos^2\alpha - \sin^2\alpha$$

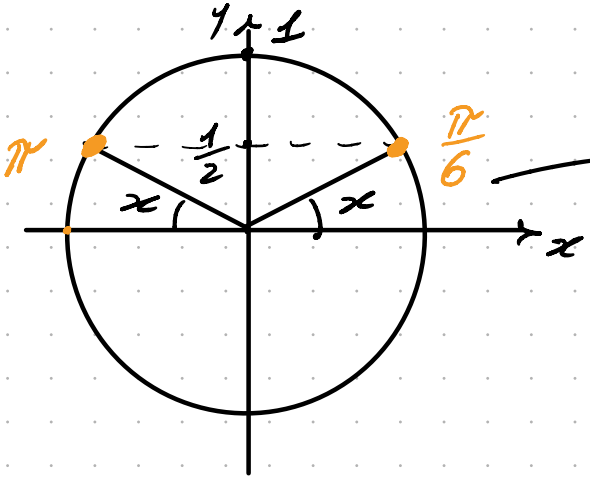
- $\cos\left(\frac{2}{3}\pi\right) = \cos\left(-\frac{2}{3}\pi\right)$ $\Longrightarrow$ VERA



$$\cos(-\theta) = \cos(\theta)$$

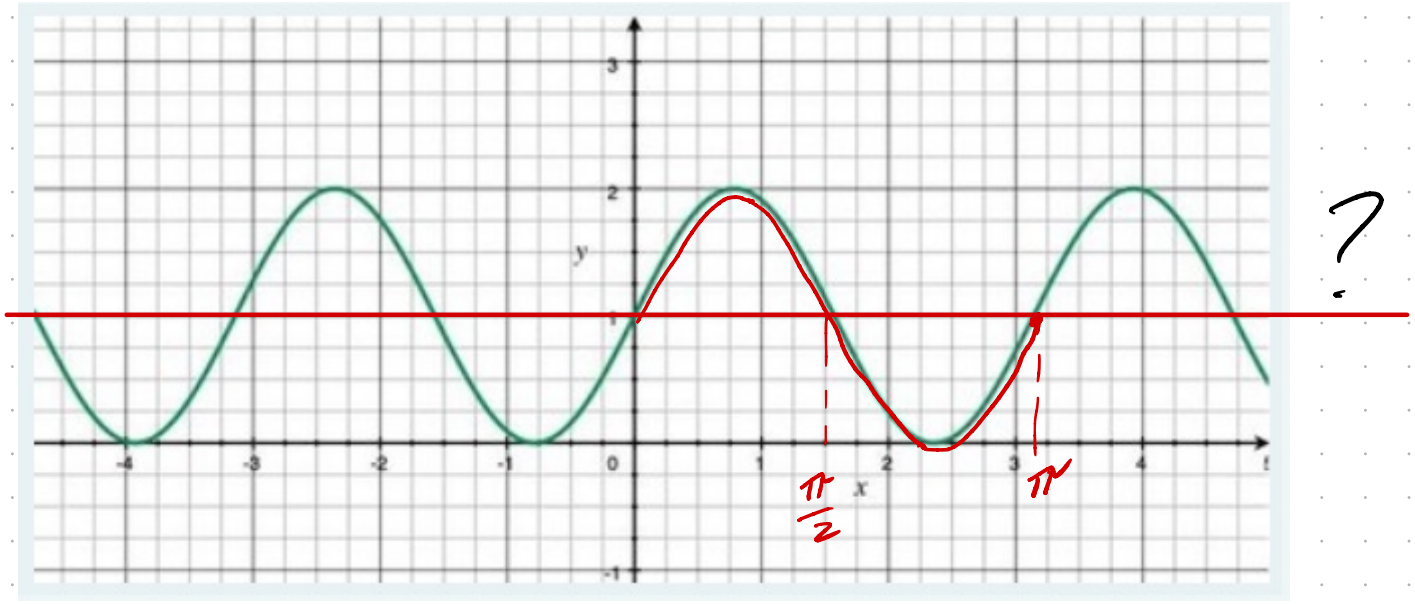
$$\Rightarrow \cos\left(\frac{2}{3}\pi\right) = \cos\left(-\frac{2}{3}\pi\right) \quad \checkmark$$

- $\sin(x) = \frac{1}{2}$ per quali $x \in [0, 2\pi]$?



$\pi - \frac{\pi}{6} = \frac{5}{6}\pi$ $\frac{\pi}{6}$ → 2 soluzioni

- $\sin(x) = 3$ → 0 soluzioni $-1 \leq \sin(x) \leq 1 \quad \forall x$

- 

?

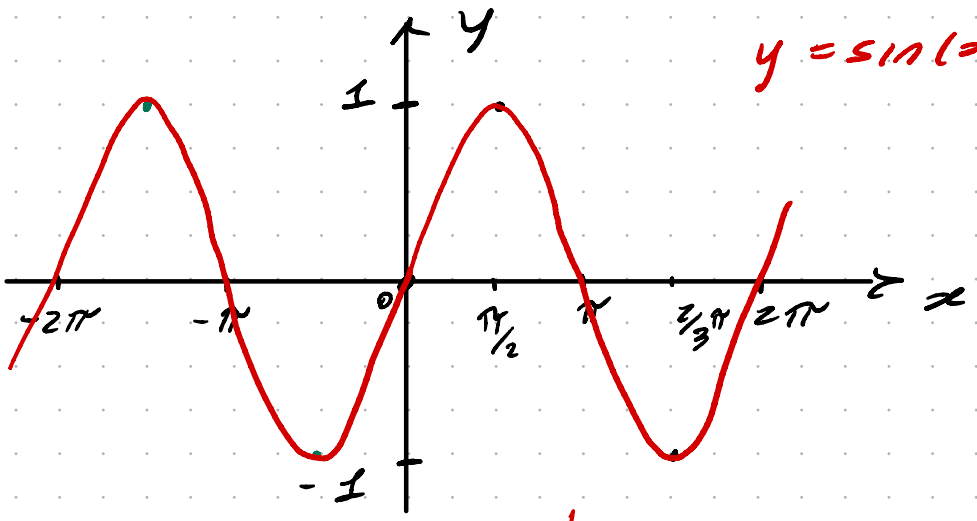
$0 \leq y \leq 2$

$y = \sin(2x) + 1$

$x = 0 \rightarrow y = 1 \quad (\sin(0) = 0)$

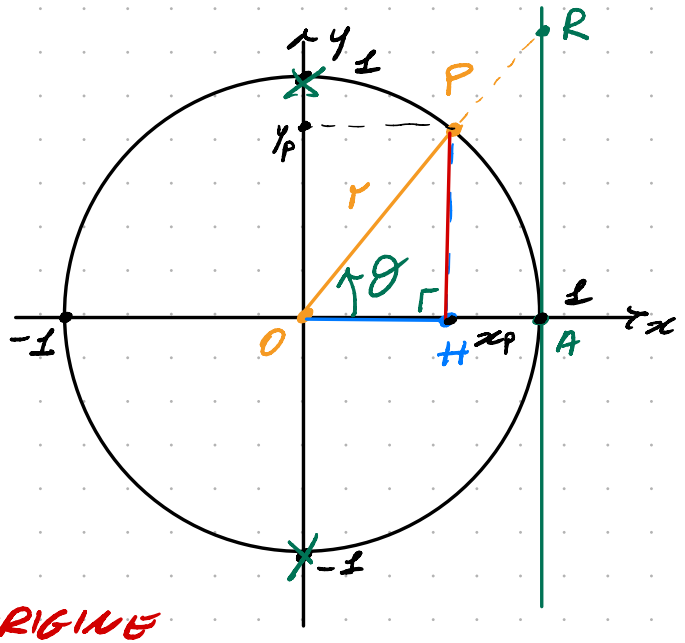
$x = \frac{\pi}{2} \rightarrow y = \sin\left(2\frac{\pi}{2}\right) + 1 = \sin(\pi) + 1 = 1$

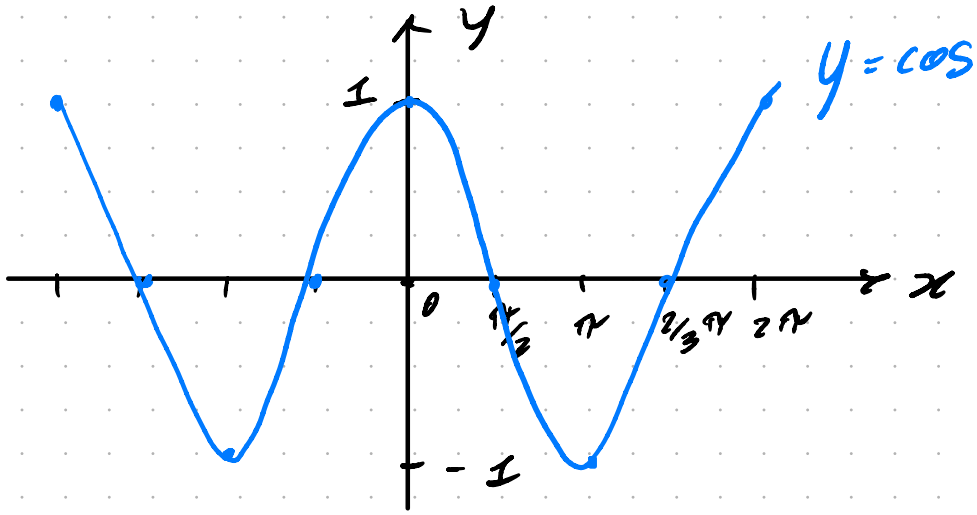
$x = \pi \rightarrow y = \sin(2\pi) + 1 = 1$ ($\sin(2\pi) = 0$)



$y = \sin(x)$

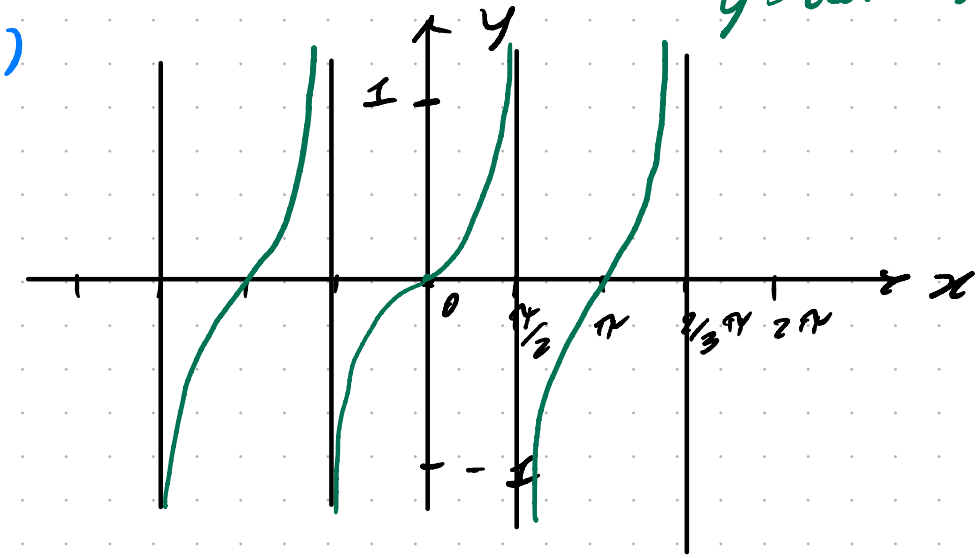
↳ DISPARI
SIMMETRICA RISPETTO L'ORIGINE



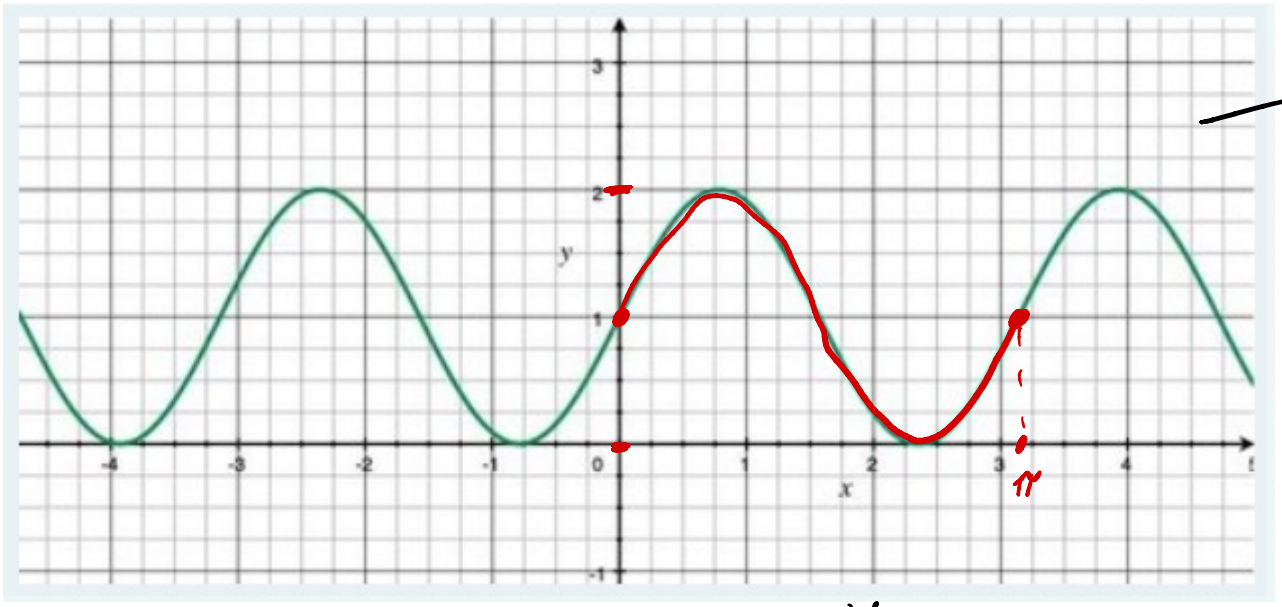


$y = \cos(x)$

↳ PARI
SIMMETRICA RISPETTO ALL'ASSE DELLE Y



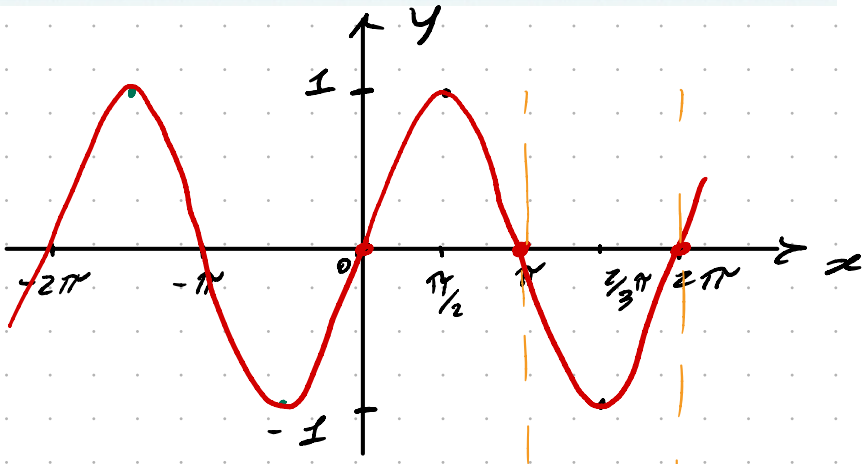
$y = \tan(x)$



$y = \sin(2x) + 1$

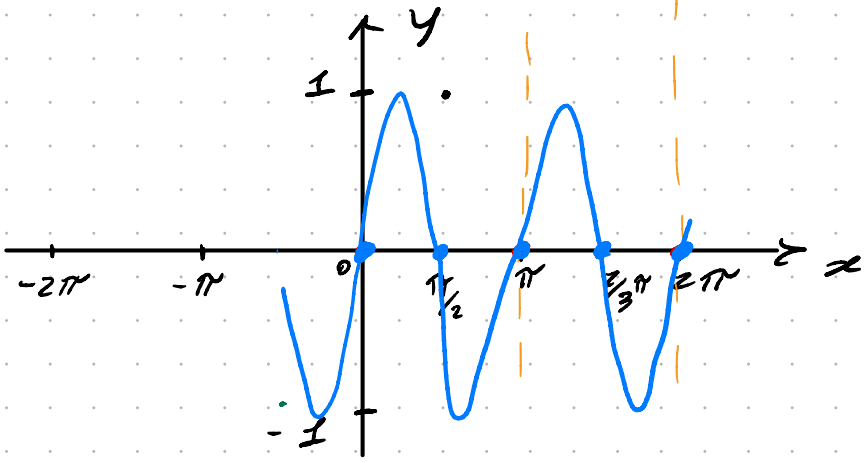
- simmetrica rispetto all'origine ⇓ dispari
- periodo π ⇒ $\sin(2x)$
- codominio $[0, 2]$ ⇒ $\sin(2x) + 1$

$y = \sin(x)$



PERIODO 2π

$y = \sin(2x)$



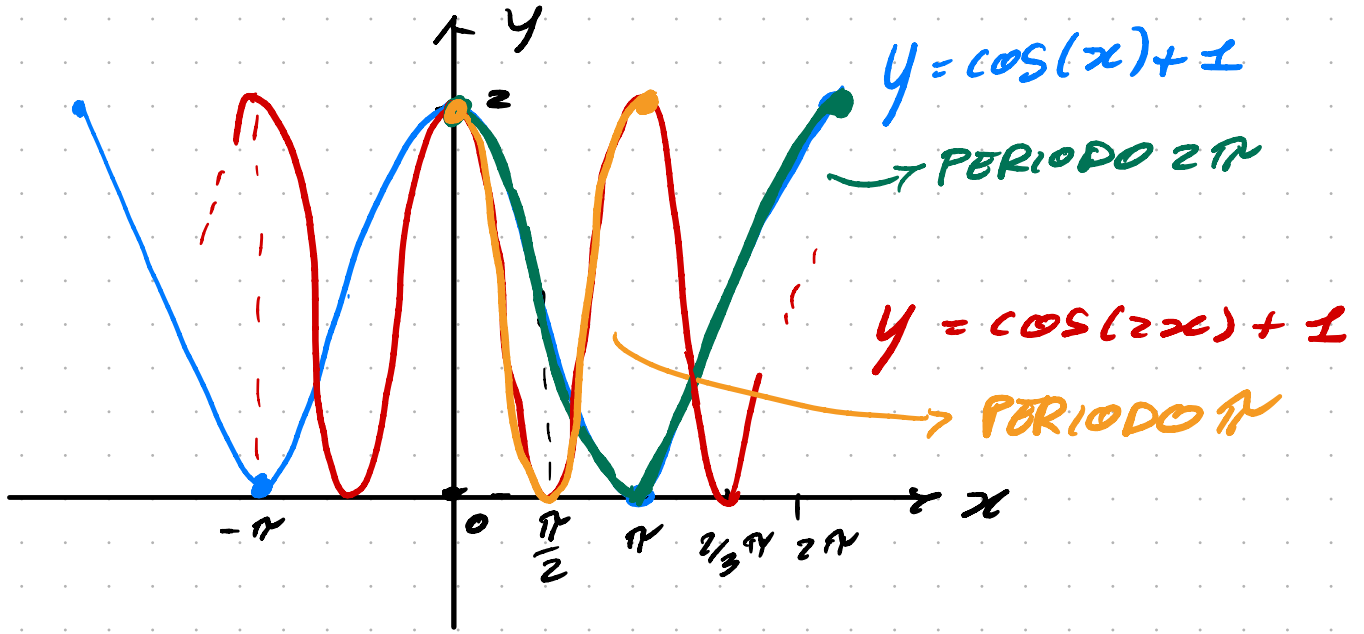
PERIODO π

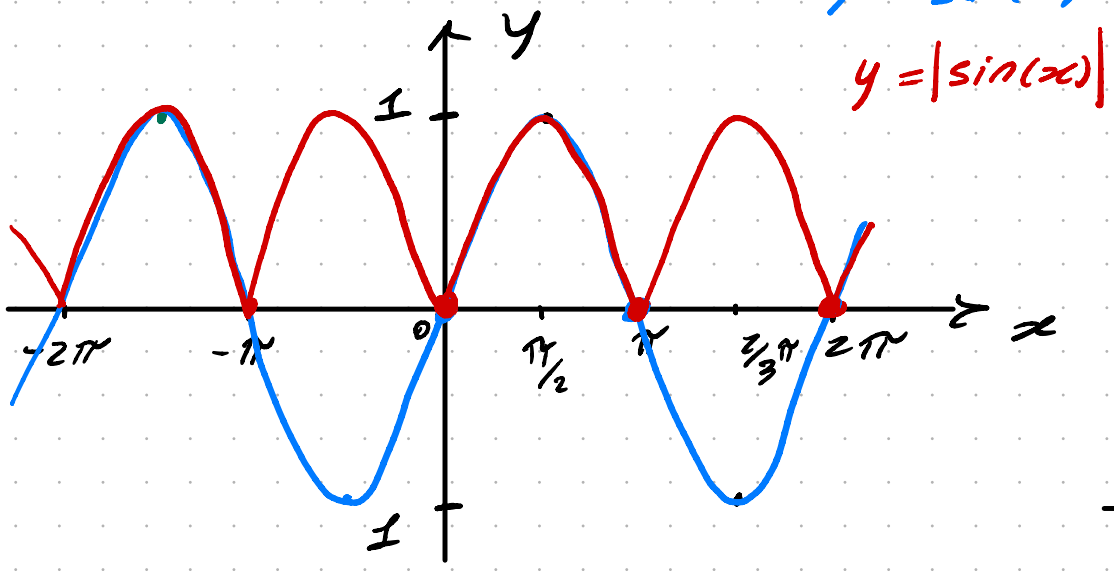
$y = \cos(2x) + 1$ ✗

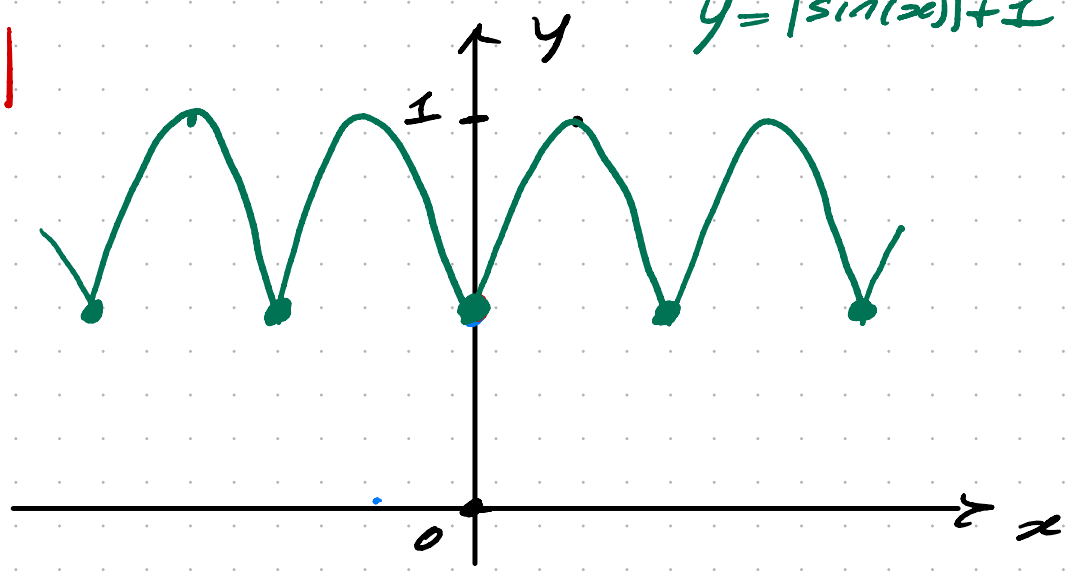
$y = \sin(2x) + 1$ ✓

$y = |\sin(x)| + 1$ ✗

$y = \cos(x) + 1$ ✗

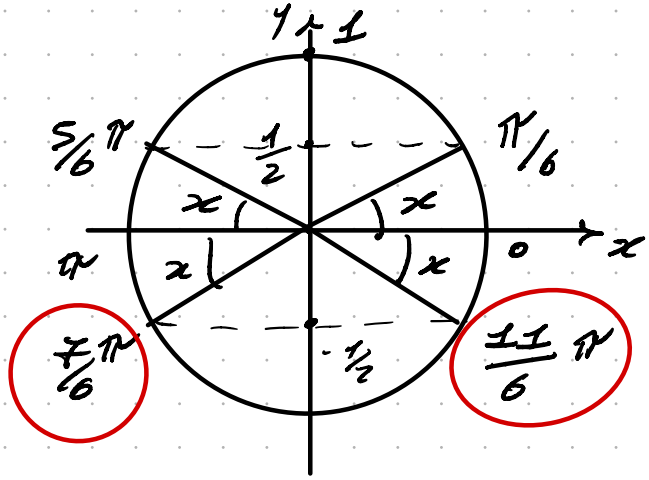






- $\sin(x) = -\frac{1}{2}$

Sol in $[0, 2\pi]$



$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

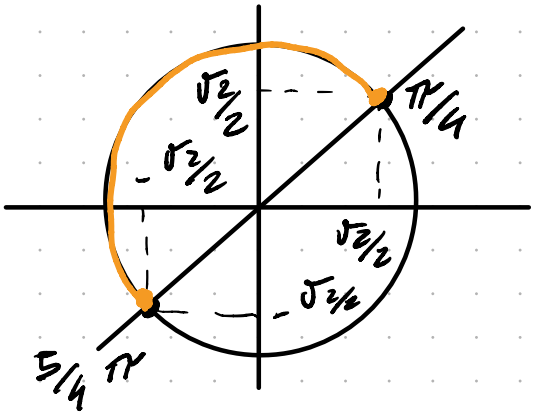
$\hookrightarrow -\sin\left(\frac{\pi}{6}\right) = -\frac{1}{2}$

$-\sin(\theta) = \sin(\pi - \theta)$

$\overset{!}{=} \sin(-\theta)$

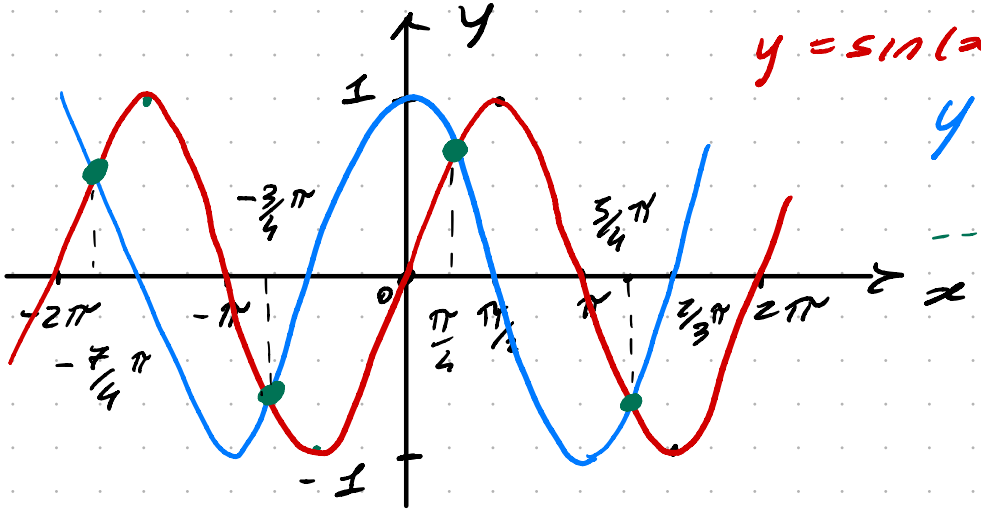
- $\sin x = \cos x$

Quante sol in $\mathbb{R}$?



$x = \left(\frac{\pi}{4} + 2k\pi\right) \vee \left(\frac{5}{4} + 2k\pi\right) \quad k \in \mathbb{Z}$

$\hookrightarrow x = \frac{\pi}{4} + k\pi \quad k \in \mathbb{Z}$



$y = \sin(x)$

$y = \cos(x$

- $\tan x = 0$ sol in $[0, \pi]$ $\Rightarrow$ 2 sol