LEZIONE 4 – EQUAZIONI E DISEQUAZIONI LOGARITMICHE E ESPONENZIALI PG. 2

- $4^{4-x} \cdot 4^{x+7} = 4^{x^2-4x-10}$

$\hookrightarrow 4^{(4-x)+(x+7)} = 4^{x^2-4x-10}$ $\longrightarrow$ STESSA BASE

$\Rightarrow 4 - \cancel{x} + \cancel{x} + 7 = x^2 - 4x - 10$

$x^2 - 4x - 21 = 0 \longrightarrow (x-7)(x+3) = 0 \longrightarrow \boxed{x_1 = -3}$, $\boxed{x_2 = 7}$

oppure

$x_{1,2} = \dfrac{4 \pm \sqrt{16+84}}{2} = \begin{cases} -3 \\ 7 \end{cases}$

- $8 + 2^{x+1} = 2^{2x}$

$\hookrightarrow 2^3 + 2^{x+1} = 2^{2x}$

$2^3 + 2^x \cdot 2 = (2^x)^2$ $\boxed{y = 2^x}$

$8 + 2y = y^2 \Leftrightarrow y^2 - 2y - 8 = 0 \Leftrightarrow (y-4)(y+2) = 0$

$\Rightarrow y_1 = -2 \ \vee \ y_2 = 4 \xrightarrow{} 2^x = -2 \ , \ 2^x = 4$

$y = 2^x$

$2^x = -2$: NO sol, $2^x > 0 \ \forall x$

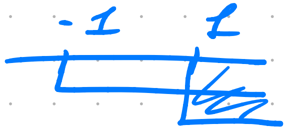
$2^x = 4$: $\boxed{x = 2}$

- $\log(x+1) = \log(2x-2)$

ARGOMENTO LOG > 0

$\log x$ $\downarrow$ base sottointesa $=$ base 10 (LOGARITMI DECIMALI)

CE $\rightarrow$ $\begin{cases} x+1>0 \\ 2x-2>0 \end{cases}$ $\begin{cases} x>-1 \\ x>1 \end{cases}$



$\underline{\underline{x>1}}$

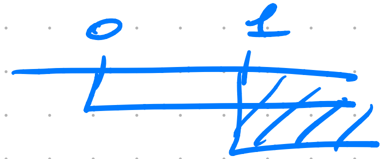
$\log(x+1) = \log(2x-2)$ STESSA BASE

$\ln x$ $\downarrow$ base e (LOGARITMI NATURALI)

$\Rightarrow \quad x+1 = 2x-2 \quad \Rightarrow \quad \boxed{x=3} > 1 \rightarrow$ OK!

- $\log x - \log 3 = \log(x-1) + \log 3$

CE $\begin{cases} x>0 \\ x-1>0 \end{cases} \rightarrow \begin{cases} x>0 \\ x>1 \end{cases} \Rightarrow$



$\underline{\underline{x>1}}$

$\log\left(\frac{x}{3}\right) = \log(3x-3)$

$\cancel{3}\cdot \frac{x}{\cancel{3}} = (3x-3)\cdot 3 \longrightarrow x = 9x-9$

$\hookrightarrow \quad 8x = 9 \rightarrow \boxed{x = \frac{9}{8}}$

PROPRIETÀ

$\log a + \log b = \log(a+b)$

$\log a - \log b = \log \frac{a}{b}$

$\log a^b = b \log a$

$\log \sqrt[b]{a} = \frac{1}{b} \log a$

- $\log_2 x - \log_x 2 = 0$

$$\log_x 2 = y \iff x^y = 2$$
$\hookrightarrow x > 0$
$x \neq 1$

CE $\begin{cases} x > 0 \\ x \neq 1 \end{cases}$

PROPRIETÀ

$$\log_a b = \frac{\log_c b}{\log_c a}$$

$$\log_2 x = \log_x 2$$

$$\log_2 x = \frac{\log_2 2}{\log_2 x}$$

$$\log_2 x = \frac{1}{\log_2 x}$$

$(\log x)^{-1} \neq \log x^{-1}$

$\hookrightarrow = \frac{1}{\log x}$ $\hookrightarrow = -\log x$

$$t = \log_2 x$$

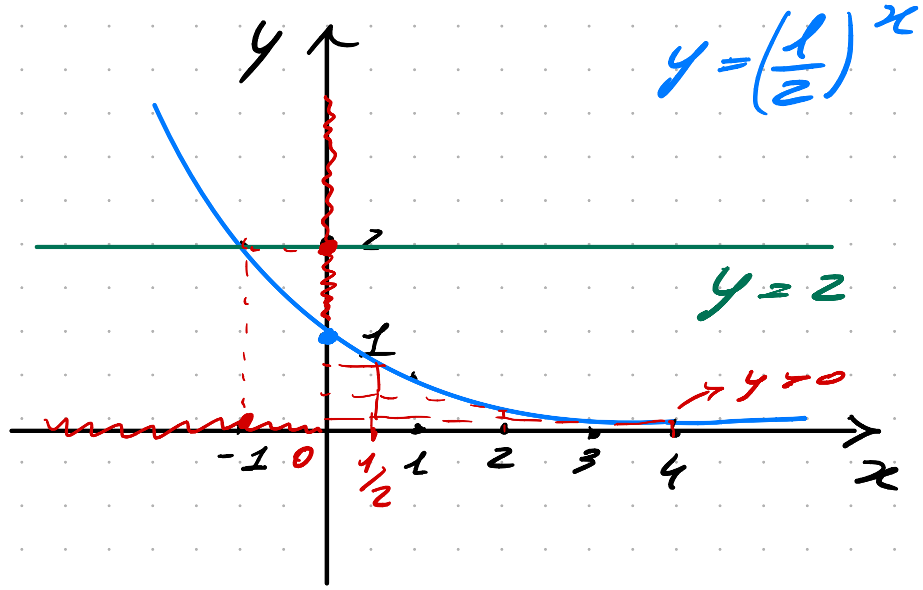
$$t = \frac{1}{t} \iff t - \frac{1}{t} = 0 \iff \frac{t^2 - 1}{t} = 0 \implies t_{1,2} = \pm 1$$

$\log_2 x_1 = 1 \rightarrow 2^1 = x_1 \implies \boxed{x_1 = 2}$ $> 0 \wedge \neq 1 \rightarrow$ OK

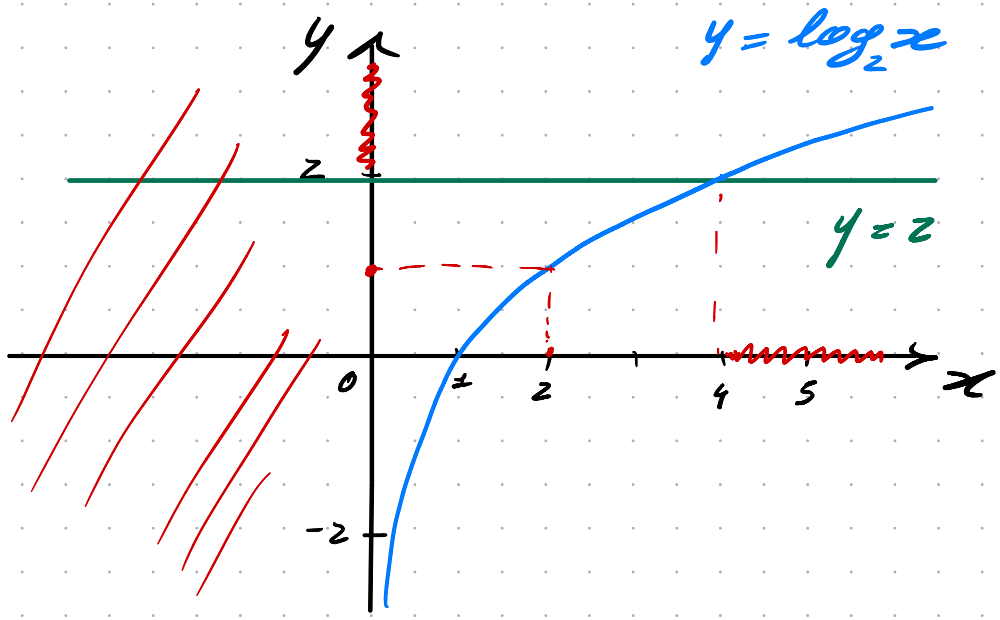
$\log_2 x_2 = -1 \rightarrow 2^{-1} = x_2 \implies \boxed{x_2 = 1/2}$ $> 0 \wedge \neq 1 \rightarrow$ OK

$\boxed{a^x = b}$ $\rightarrow \log a^x = \log b \rightarrow x \log a = \log b$

$$\rightarrow x = \frac{\log b}{\log a}$$



- $\left(\frac{1}{2}\right)^4 < 0 \rightarrow y > 0 \ \forall x$
  $\Downarrow$
  impossibile
- $\left(\frac{1}{2}\right)^{1/2} > 1 \rightarrow y > 1 \ \forall x < 0$
  $\Downarrow$
  impossibile
- $\boxed{\left(\frac{1}{2}\right)^2 < 2} \rightarrow$ OK!
- $\left(\frac{1}{2}\right)^{-1} > 2 \rightarrow \left(\frac{1}{2}\right)^{-1} = 2 \rightarrow$ NO > 2



- $\log_2 2 > 2 \rightarrow$ NO!!!
- $\log(-2) > 0 \rightarrow$ ARGOMENTO DEL LOG > 0
  $\downarrow$ NON ACC.
  $\Rightarrow$ impossibile
- $\log(5) < 2 \rightarrow \forall x > 4 \rightarrow y > 2$
  $\Rightarrow$ NO!!!
- $\boxed{\log(5) > 2} \rightarrow$ SI!!!

- $e^{\ln z} = z$ $\quad$ $\ln z$ è l'esponente che occorre ad e per ottenere z

$$a^{\log_a b} = b$$

- $\log_7 140 = y \rightarrow 7^y = 140$

$$\log_7 140 = \log_7 (10 \cdot 14) = \log_7 (2 \cdot 5 \cdot 2 \cdot 7) = \log_7 (2^2 \cdot 5 \cdot 7)$$
$$= \log_7 2^2 + \log_7 5 + \log_7 7$$
$$= \log_7 4 + \log_7 5 + 1$$
$$= 1 + \log_7 (4 \cdot 5) = \underline{1 + \log_7 20}$$

- $y = 20 \gg 140 \rightarrow$ NO!!!
- $3 < y < 7 \rightarrow 7^3 = 343 > 140$ $\hookrightarrow$ NO!!!
- $y > 7 \rightarrow$ NO!
- $\boxed{y = 1 + \log_7 20} \rightarrow$ OK!

- $\left(\frac{\sqrt{2}}{2}\right)^x < 2 \rightarrow (2^{1/2} \cdot 2^{-1})^x < 2^1$

  $(2^{-1/2})^x < 2^1$

  $-\frac{1}{2}x < 1 \rightarrow -x < 2 \rightarrow \boxed{x > -2}$

- $\log_{3/4}(x) = -3 \rightarrow x = (3/4)^{-3} = \frac{4^3}{3^3} = \frac{64}{27}$

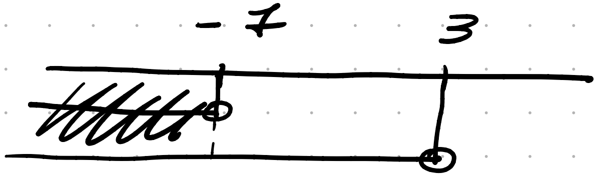
  ↳ $x > 0$

  ↳ $\boxed{x = \frac{64}{27}}$

- $\log_{10}(3-x) > 1 \rightarrow$ CE $\quad 3 - x > 0 \Rightarrow x < 3$

  $\log_{10}(3-x) > \underbrace{\log_{10} 10}_{=1} \Rightarrow$ STESSA BASE

  $3 - x > 10 \rightarrow x < -7 \Rightarrow$



  ↳ $\boxed{x < -7}$

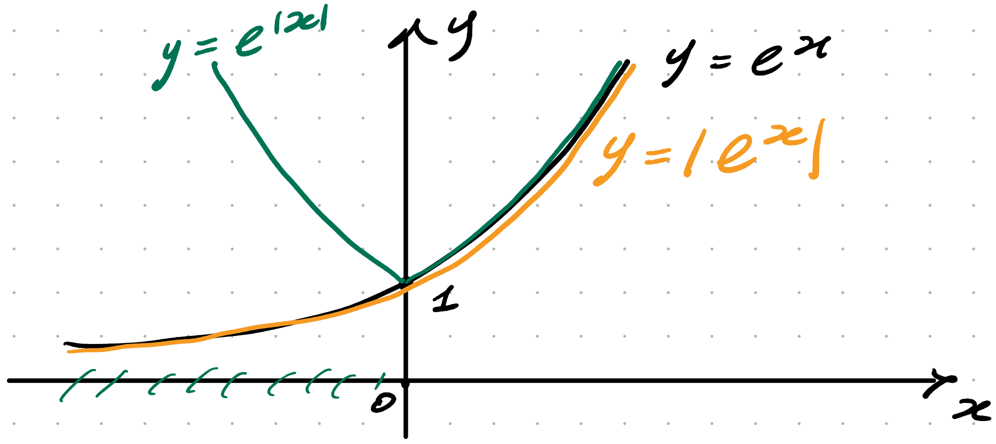
GRAFICI

$y = e^x$

$y = e^{|x|}$

$y = |e^x| = e^x$

$\underbrace{}_{\substack{>0 \\ \text{sempre}}}$

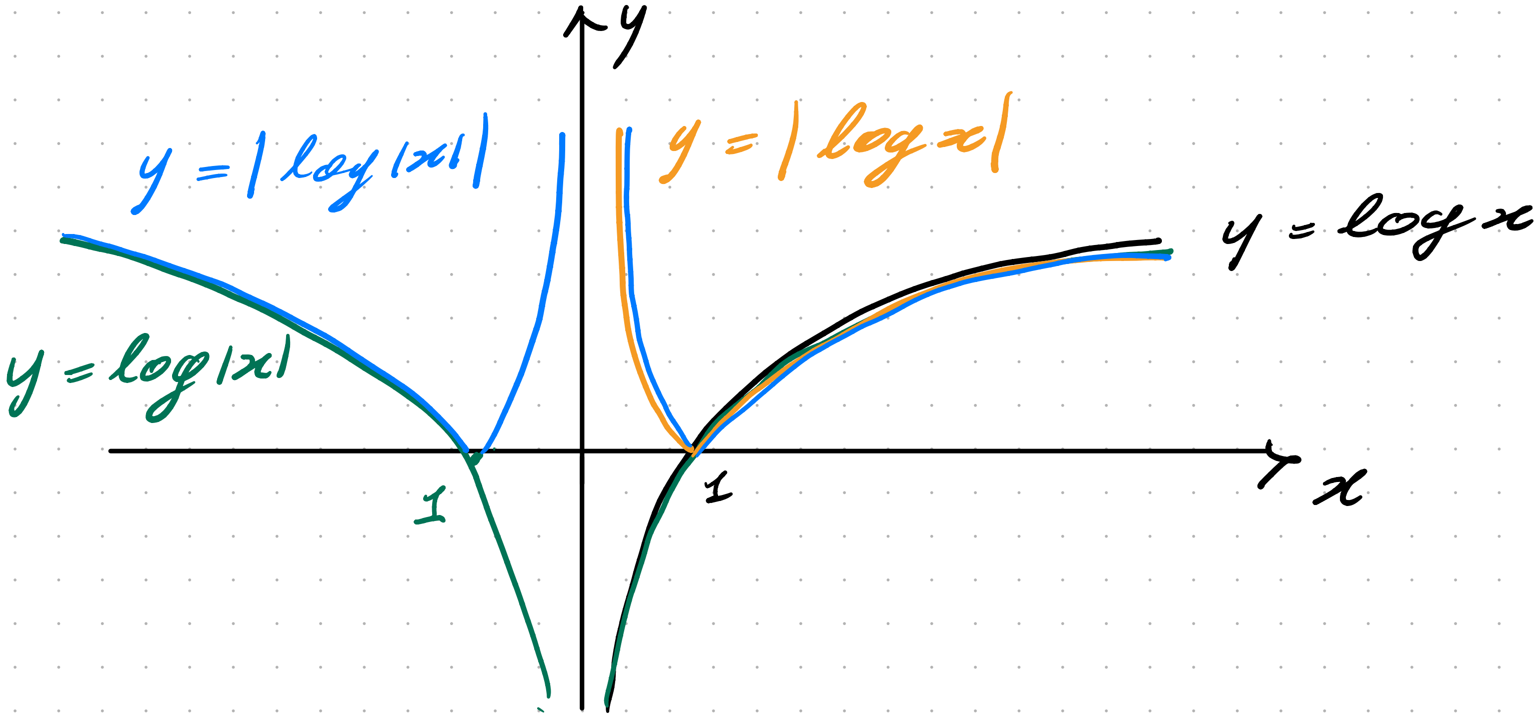


$y = \log x$

$y = \log|x|$

$y = |\log x|$

$y = |\log|x||$



Ex

- $\log_2 3 > \log_2 x \xrightarrow{\text{STESSA BASE}} \begin{cases} x > 0 \\ x < 3 \end{cases} \Rightarrow \boxed{0 < x < 3}$

- $\log_2(x-3) < 2 \rightarrow x - 3 > 0 \rightarrow x > 3$

$\log_2(x-3) < 2 \log_2 2 \rightarrow \log(x-3) < \log 2^2$

$x - 3 < 4 \Rightarrow x < 7 \Rightarrow \begin{cases} x > 3 \\ x < 7 \end{cases} \Rightarrow \boxed{3 < x < 7}$

- $\left(\frac{1}{2}\right)^x > 4 \rightarrow (2^{-1})^x > 2^2$ oppure $\left(\frac{1}{2}\right)^x > \left(\frac{1}{2}\right)^{-2}$

$\Rightarrow \boxed{x > -2}$

- $2^{x+1} \geq 6 \rightarrow \log_2 2^{x+1} \geq \log_2 6$

$x+1 \underbrace{\log_2 2}_{=1} \geq \log_2 6$

$x+1 \geq \log_2 6 \Rightarrow \boxed{x \geq \log_2 6 - 1}$