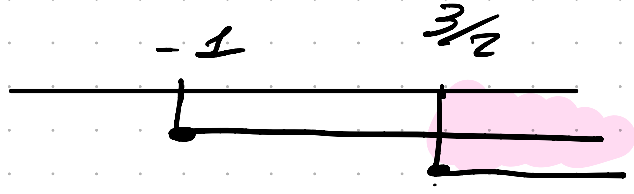
LEZIONE 3

- $\sqrt{x-1} - \sqrt{2x-3} = 0 \Rightarrow \sqrt{x-1} = \sqrt{2x-3}$

CE $\rightarrow \begin{cases} x+1 \geq 0 \\ 2x-3 \geq 0 \end{cases} \rightarrow \begin{cases} x \geq -1 \\ x \geq 3/2 \end{cases} \rightarrow$



$x \geq 3/2$

$(\sqrt{x-1})^2 = (\sqrt{2x-3})^2$

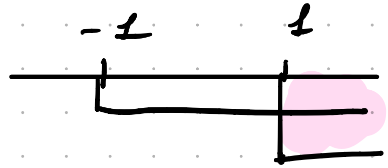
$x - 1 = 2x - 3 \Rightarrow \boxed{x = 2} \geq \frac{3}{2} \rightarrow$ OK!

- $\sqrt[3]{x+4} = 3 \rightarrow (\sqrt[3]{x+4})^3 = (3)^3$

$\Rightarrow x + 4 = 27 \rightarrow \boxed{x = 23} \rightarrow$ OK!

- $\sqrt{x^2+3} < x+1 \rightarrow \begin{cases} x^2+3 \geq 0 \rightarrow \text{sempre} \\ x+1 > 0 \\ x^2+3 < (x+1)^2 \end{cases}$

$\Rightarrow \begin{cases} x > -1 \\ \cancel{x^2} + 3 < \cancel{x^2} + 2x + 1 \end{cases} \rightarrow \begin{cases} x > -1 \\ 2x - 2 > 0 \end{cases} \rightarrow \begin{cases} x > -1 \\ x > 1 \end{cases} \rightarrow$

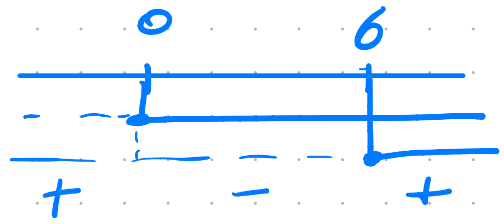


$\boxed{x > 1}$

- $\sqrt{x^2-6x} > x+2$

$$\begin{cases} x^2-6x \geq 0 \\ x+2 \leq 0 \end{cases} \cup \begin{cases} x^2-6x \geq 0 \\ x+2 > 0 \\ x^2-6x > (x+2)^2 \end{cases}$$

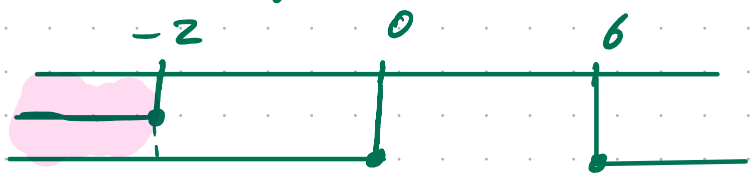
$$\begin{cases} x(x-6) \geq 0 \\ x+2 \leq 0 \end{cases} \cup \begin{cases} x(x-6) \geq 0 \\ x > -2 \\ \cancel{x^2} - 6x > \cancel{x^2} + 4x + 4 \end{cases}$$

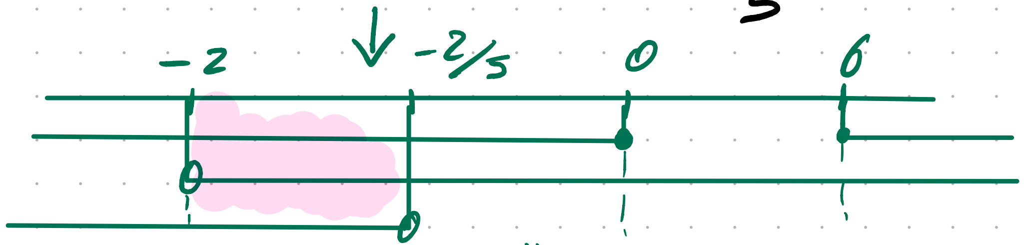


$x \leq 0 \vee x \geq 6$

$$\begin{cases} x \leq 0 \vee x \geq 6 \\ x \leq -2 \end{cases} \cup \begin{cases} x \leq 0 \vee x \geq 6 \\ x > -2 \\ 10x + 4 < 0 \rightarrow x < -\frac{2}{5} \end{cases}$$

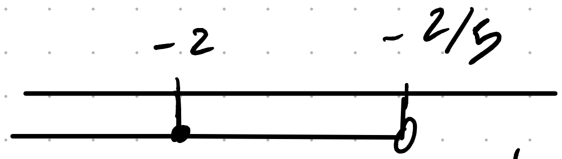
$\downarrow$ $\downarrow$





$\Downarrow$ $\Downarrow$

$x \leq -2$ $-2 < x < -2/5$



$\boxed{x < -2/5}$

EQUAZIONI LOGARITMICHE ED ESPONENZIALI

- FUNZIONE POTENZA $\to x^n$ n fissato

FUNZIONE ESPONENZIALE $\to a^x$ a fissato, $x \in \mathbb{R}$

intero: $a^n = \underbrace{a \cdot \ldots \cdot a}_{n \text{ volte}}$ | relativo: $a^{-n} = \frac{1}{a^n}$ | razionale: $a^{m/n} = \sqrt[n]{a^m}$

n pari $\to \sqrt[n]{-2} \notin \mathbb{R}$

$$\Downarrow \qquad \Downarrow$$

$$\underbrace{a \neq 0 \qquad a \geq 0}$$

$$\Downarrow$$

Avendo $x \in \mathbb{R} \Rightarrow a > 0$

$a = 1 \Rightarrow a^1 = a^2 = \ldots = a^n = 1 \to$ non ha significato

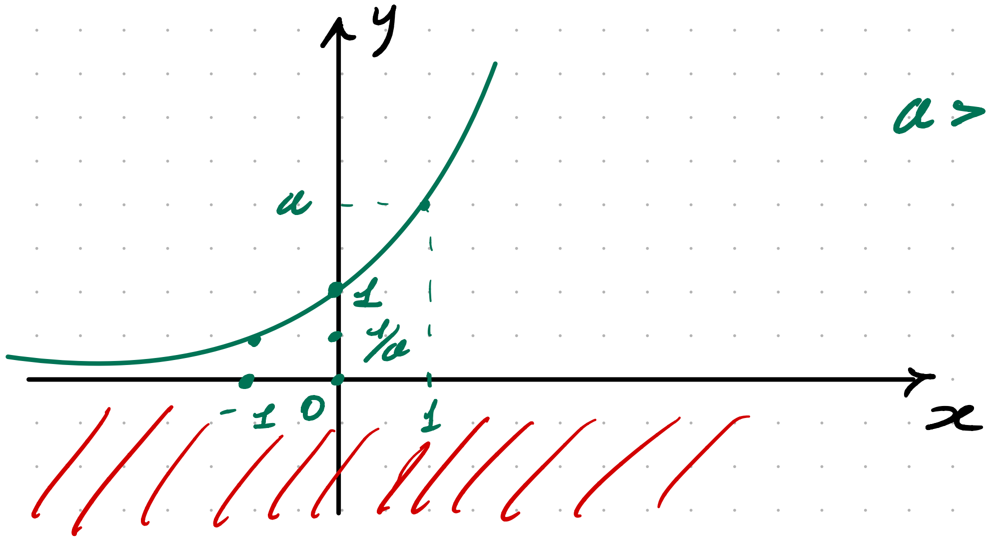
$$\Downarrow$$

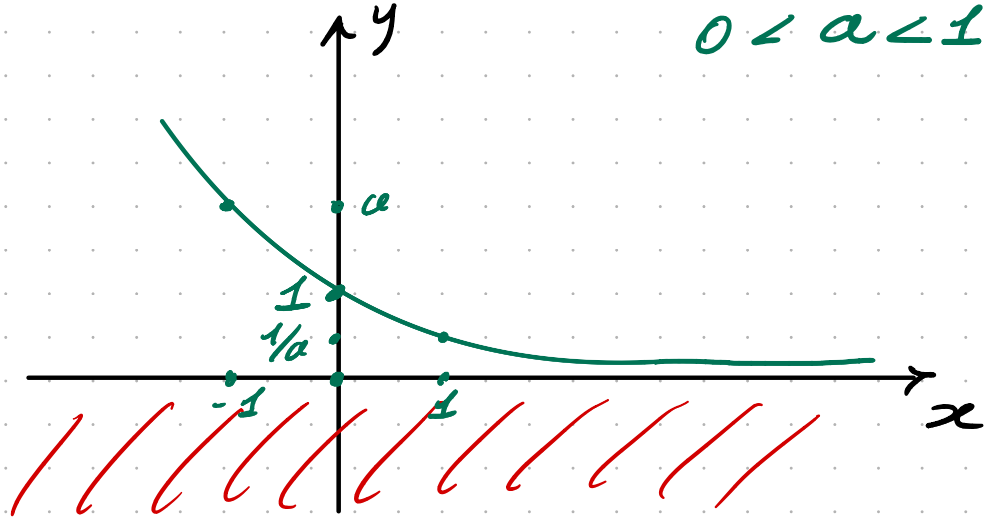
$$a \neq 1$$

RIASSUMENDO:

a^x :
- $x \in \mathbb{R}$, è definito per ogni valore di x
- $a > 0 \wedge a \neq 1$, è definito solo per basi positive
- $a^x > 0$, ha valori sempre positivi
- $a^x = 1$ se $x = 0$

$y = a^x \qquad a > 0 \quad , a \neq 1$





$\Rightarrow$ L'ESPONENZIALE NON INTERSECA MAI L'ASSE x !!!

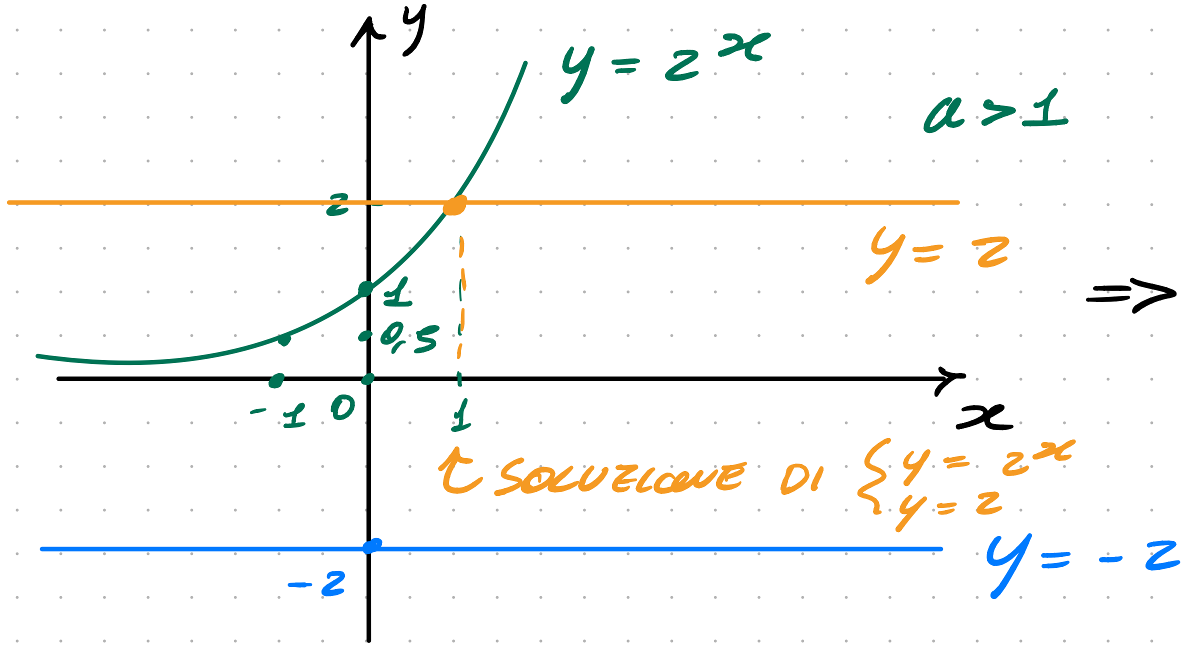
- $e^x = 0 \rightarrow$ NON AMMETTE SOLUZIONI ($e \approx 2,7$) ↳ Numero di Nepero

  ↓

  Vale lo stesso per $2^x = 0$, $10^x = 0$ ....

A vedere $y = a^x$, y non sarà mai $= 0$ $\forall$ valore di a

• $2^x = -2$



NO

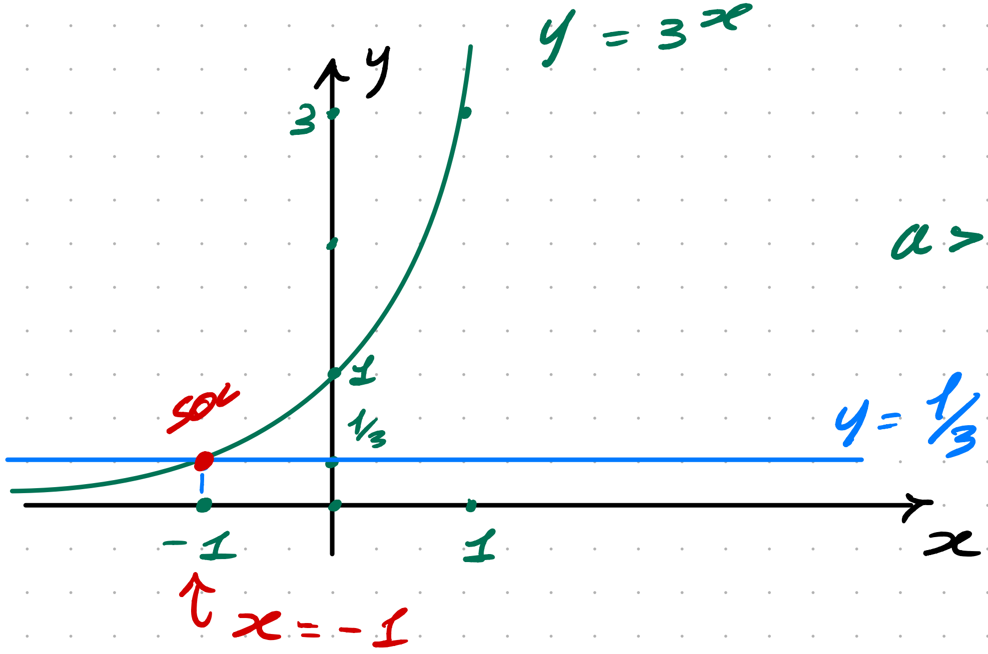
$\Rightarrow$ INTERSEZIONI $\Rightarrow \nexists$ SOL

TRA $y = 2^x$ e

$y = -2$

Se $-(2^x) = -2 \Rightarrow 2^x = 2 \Rightarrow \boxed{x = 1}$

• $3^x = \frac{1}{3} \Rightarrow \boxed{x = -1}$



$a > 1$

$$3^x = \frac{1}{3}$$
$$\hookrightarrow 3^x = 3^{-1}$$
$$\Rightarrow x = -1$$

ESPONENZIALE

$a^x = b$, $a > 0 \wedge a \neq 1 \Rightarrow b \neq 0 \wedge b > 0$

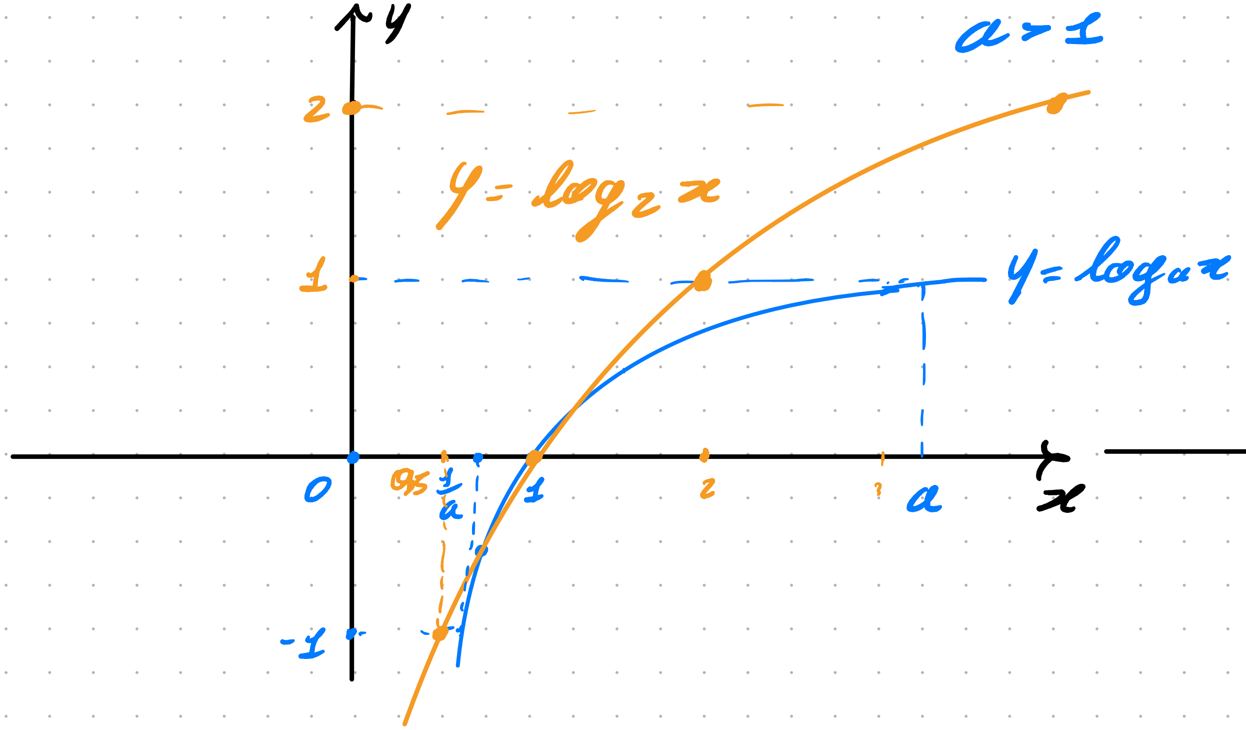
$$x = \log_a b \rightarrow \text{LOGARITMO}$$

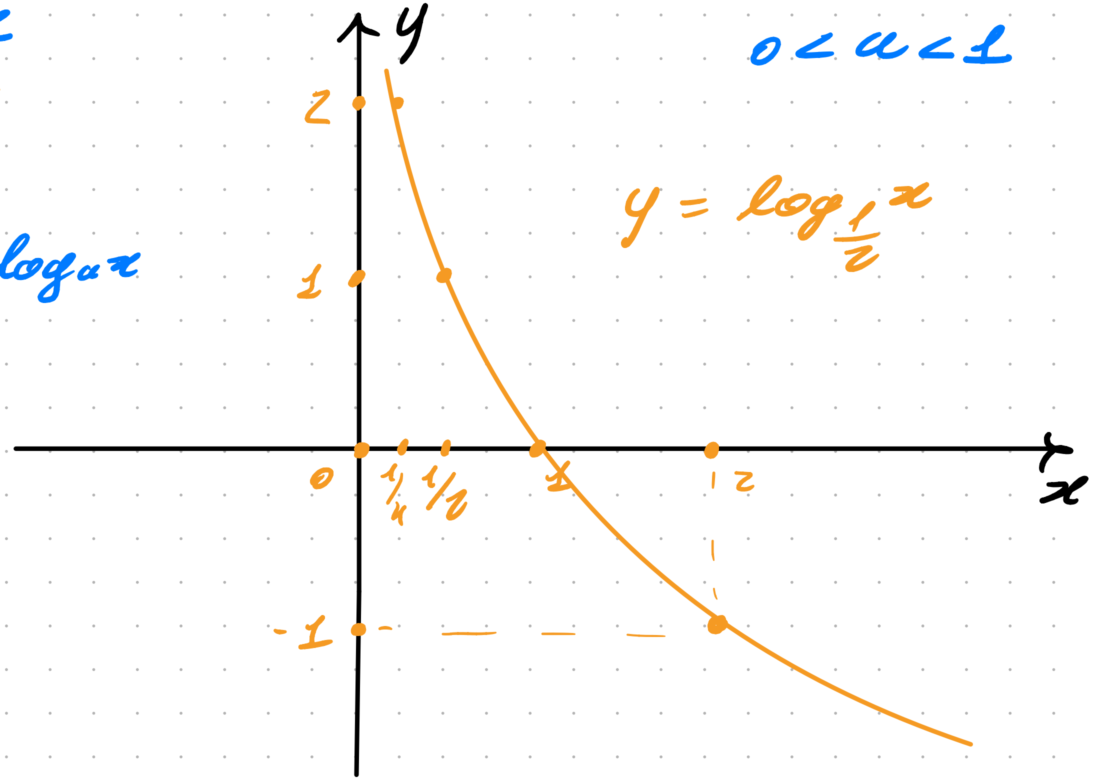
$\hookrightarrow$ esponente che devo assegnare ad a per ottenere b

$y = \log_a x \quad \rightarrow \quad a^y = x$

↳ ARGOMENTO

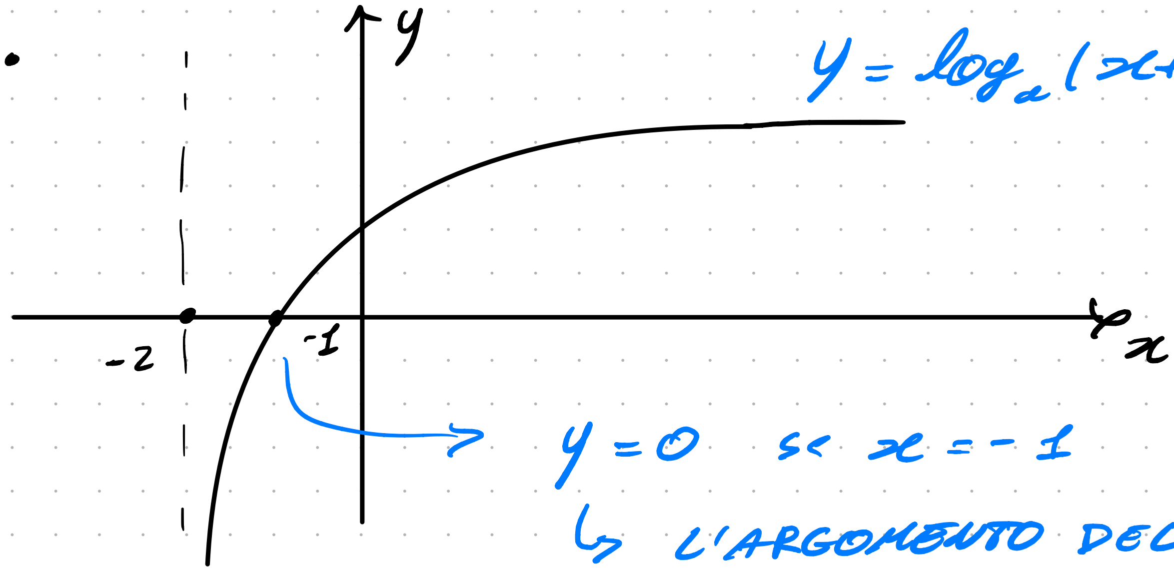
↳ VALORE DEL LOGARITMO = ESPONENTE DI a





Se $x = 1 \Rightarrow y = 0$

Infatti prendendo $a^{\overbrace{y}^{=0}} = \underbrace{x}_{=1}$ $\quad (a^0 = 1)$



$y = \log_a(x+2)$

$y = \log_a x$

$y = 0$ se $x = 1$

$y = 0$ se $x = -1$ $\Rightarrow$

↳ L'ARGOMENTO DEL LOGARITMO È $= 1$
QUANDO $x = -1$

$\Rightarrow \quad y = \log_a \hat{x} = \log_a(x+b)$

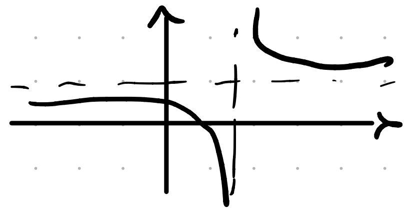
$\hat{x} = x + b \xrightarrow[\substack{\text{se } x=-1 \\ \hat{x}=1}]{} -1+b = 1 \Rightarrow \hat{x} = x+2$

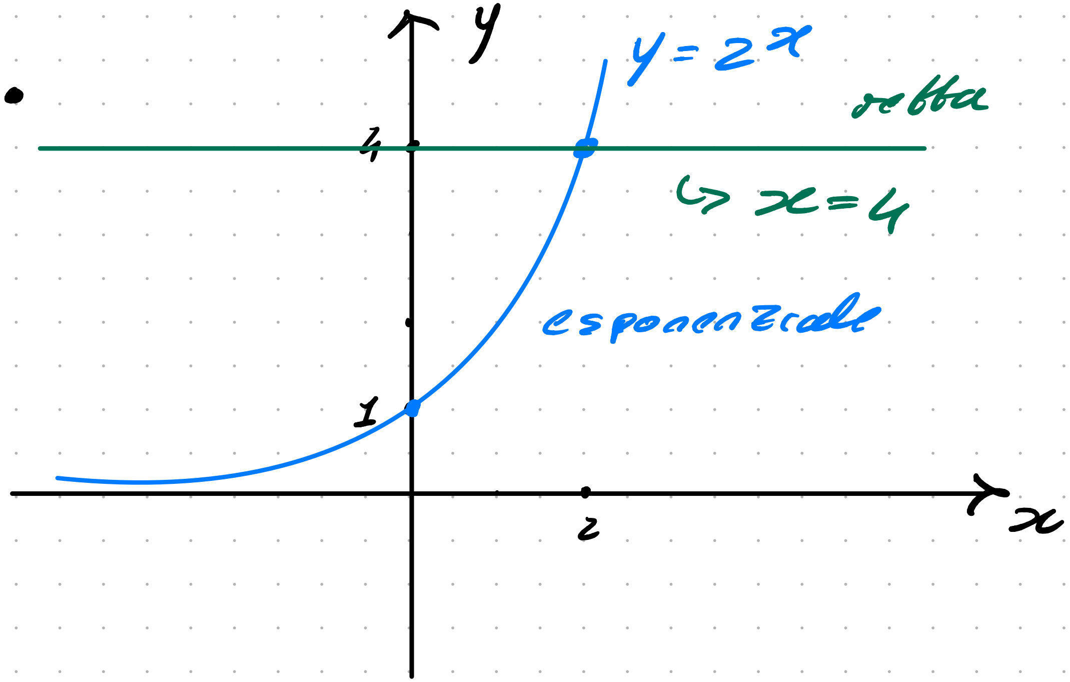
↳ $b = 2$

ALTRE RISPOSTE

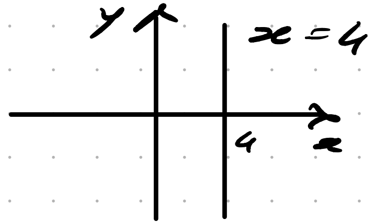
- Lineare → RETTA ⇒ NO!
- Esponenziale →  ⇒ NO!!

Nel grafico ci sono valori di $y < 0$

- Omografica → $y = \dfrac{ax+b}{cx+d}$  ⇒ NO!!!

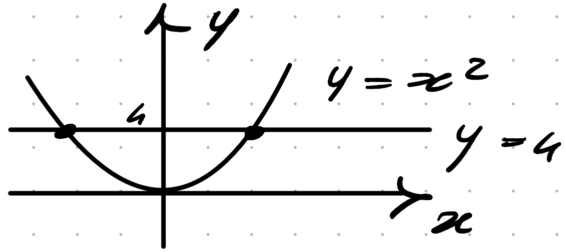
- 

$x = 4$

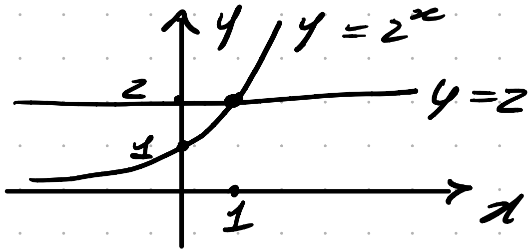


$2^x = 4 \longrightarrow$ OK!

$x^2 = 4$



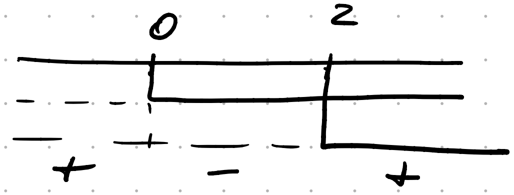
$2^x = 2$



$$\log_a x = y \iff a^y = x$$

- $\log_2 4 = x \rightarrow 2^x = 4 \Rightarrow 2^x = 2^2 \Rightarrow x = 2$
- $\log_x 3 = -1 \rightarrow x^{-1} = 3 \Rightarrow \frac{1}{x} = 3 \Rightarrow x = \frac{1}{3}$
- $\log_5 x = 2 \rightarrow 5^2 = x \Rightarrow x = 25$

Ex



- $\log_3 (x^2 - 2x) = 1$

$$x^2 - 2x > 0 \Rightarrow x(x-2) > 0 \rightarrow (x<0) \vee (x>2)$$

$$\log_3 (x^2 - 2x) = 1 \Rightarrow x^2 - 2x = 3^1 \Rightarrow x^2 - 2x - 3 = 0$$

$$x_{1/2} = \frac{2 \pm \sqrt{4+12}}{2} = \begin{cases} -1 < 0 & \text{ok!} \\ 3 > 2 & \text{ok!} \end{cases}$$

- $\log (e^x + e) = 2$ $\qquad e^x > 0 \ \forall x \Rightarrow e^x + e > 0 \ \forall x$

$$e^x + e = e^2 \Rightarrow e^x = e^2 - e = e(e-1)$$

$$\ln e^x = \ln [e(e-1)] \Rightarrow x \underbrace{\ln e}_{=1} = \underbrace{\ln e}_{=1} + \ln(e-1)$$

$$\hookrightarrow \quad x = 1 + \ln(e-1)$$