LEZIONE 2 – EQUAZIONI E DISEQUAZIONI IRRAZIONALI

quando compaiono uno o più radicali aventi nel radicando l'incognita

$\sqrt[n]{f(x)}$

- n pari ha significato $\iff f(x) \geq 0$
- se n pari, $f(x) \geq 0 \Rightarrow \sqrt[n]{f(x)} \geq 0$

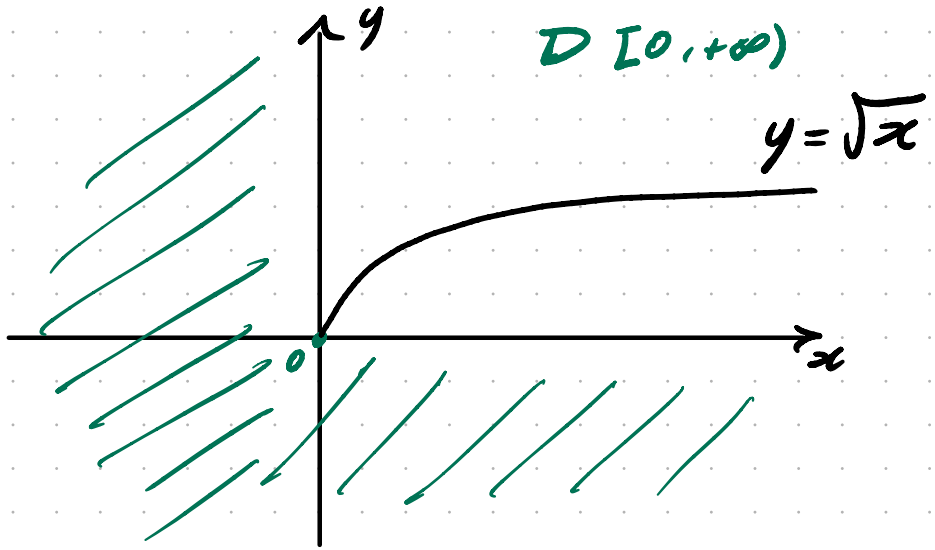
$$\sqrt[n]{f(x)} = g(x) \iff f(x) = [g(x)]^n$$

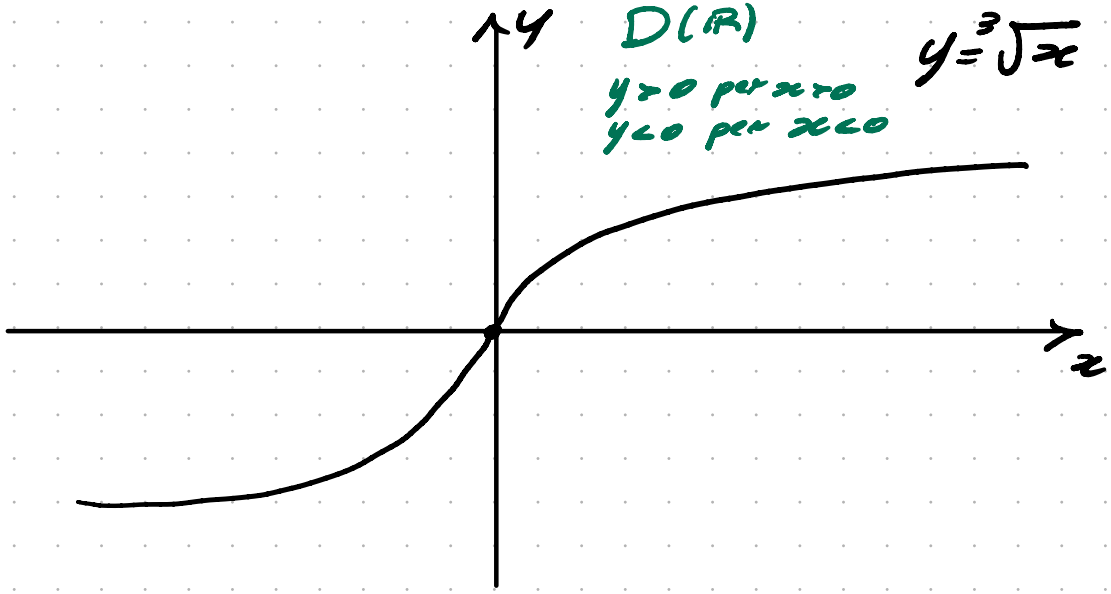
se n disp
se n pari $g(x) \geq 0$

ex

$g(x) = \sqrt[2]{x} - 1$ (2 → pari) $\Rightarrow$ RADICE PARI $\Rightarrow$ $x \geq 0$, $D\ [0, +\infty)$

$g(x) = \sqrt[3]{x} - 1 \Rightarrow$ RADICE DISPARI $\Rightarrow D(\mathbb{R})$





ES $\sqrt{x+1} = x-1$

$\rightarrow$ CE $\begin{cases} x+1 \geq 0 \rightarrow x \geq -1 \\ x-1 \geq 0 \rightarrow x \geq 1 \end{cases}$

$x+1 = (x-1)^2$

$x+1 = x^2 - 2x + 1$

$x^2 - 3x = 0$

$x(x-3) = 0$

$x = 0 \quad < 1 \rightarrow$ NON ACCETTABILE!

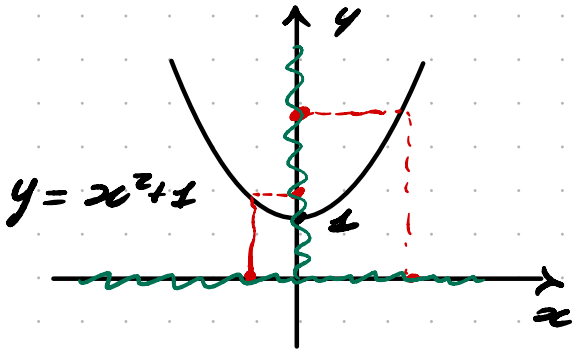
$x = 3 \quad \geq 1 \rightarrow$ OK!

SOL $\rightarrow$ $\boxed{x = 3}$

$g(x) = \sqrt[2]{x^2+1}$ → RADICE PARI ⇒ $x^2+1 \geq 0$

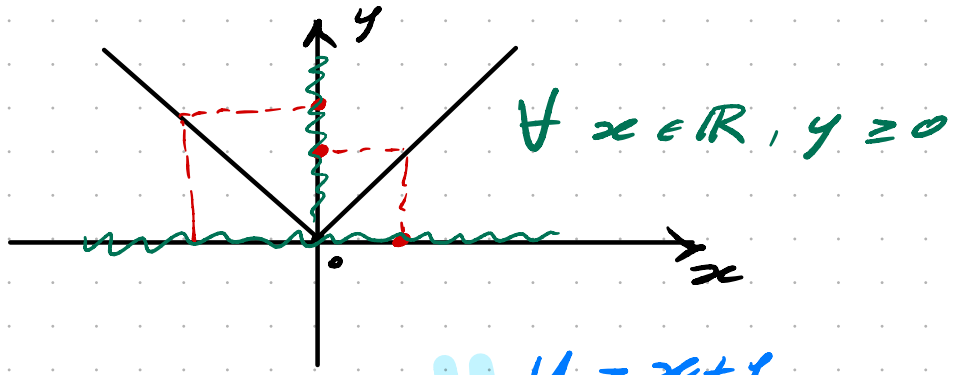
↳ $D(\mathbb{R})$

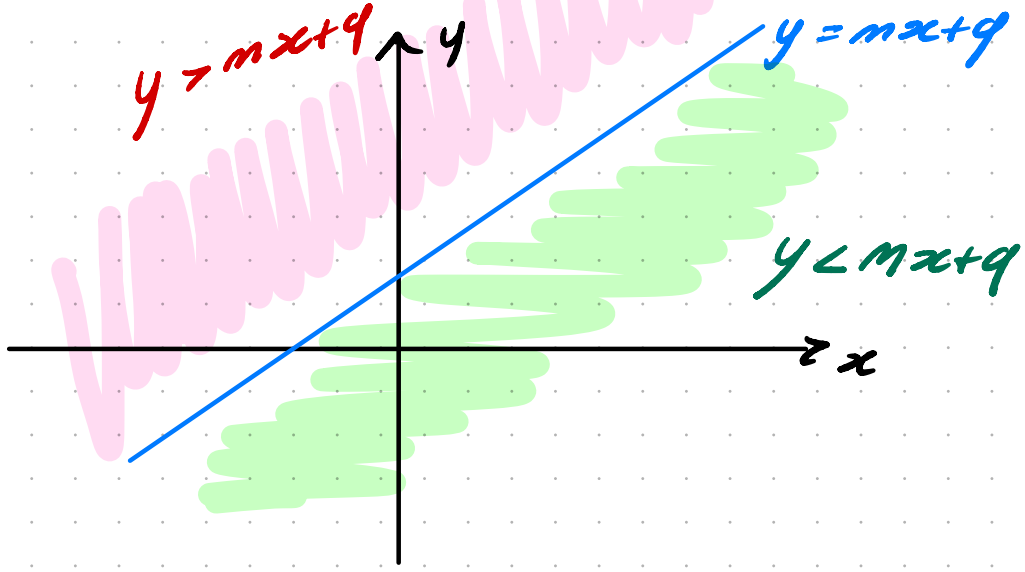
x^2+2
x^2+100 ⇒ SEMPRE POSITIVI
x^2+200

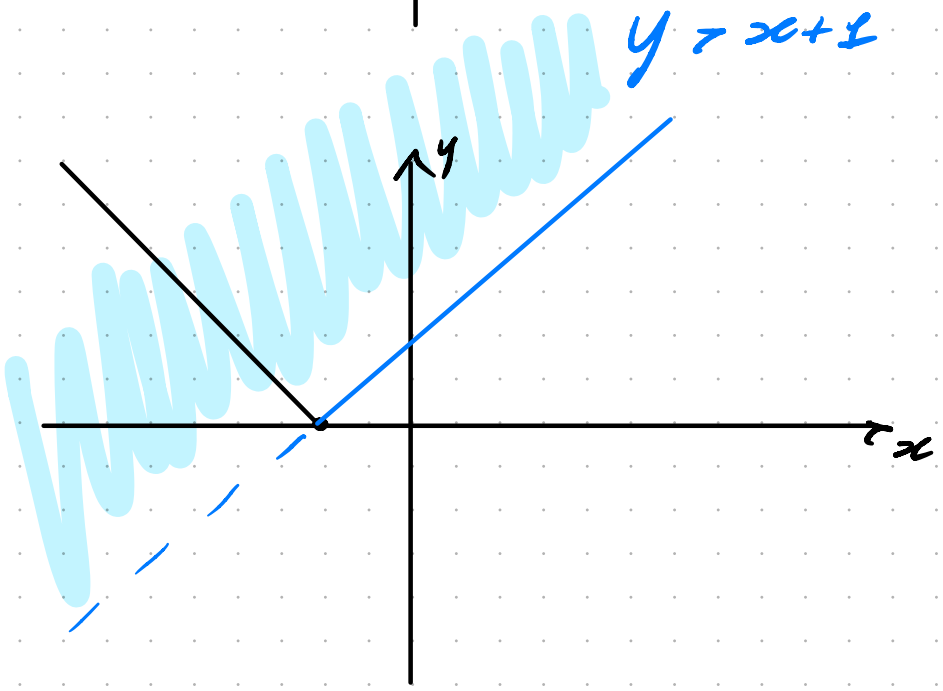


$-1 = \sqrt{x^2}$ ⇒ ∄ soluzioni
↓ <0 ; $\sqrt{x^2}$ ≥0

$$\sqrt{x^2} = |x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

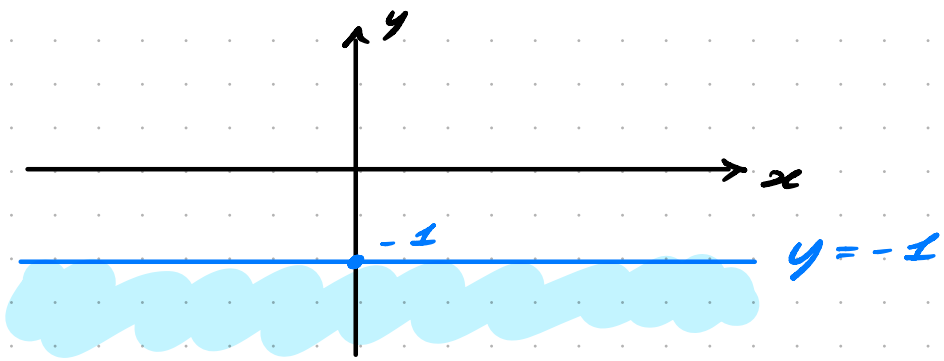




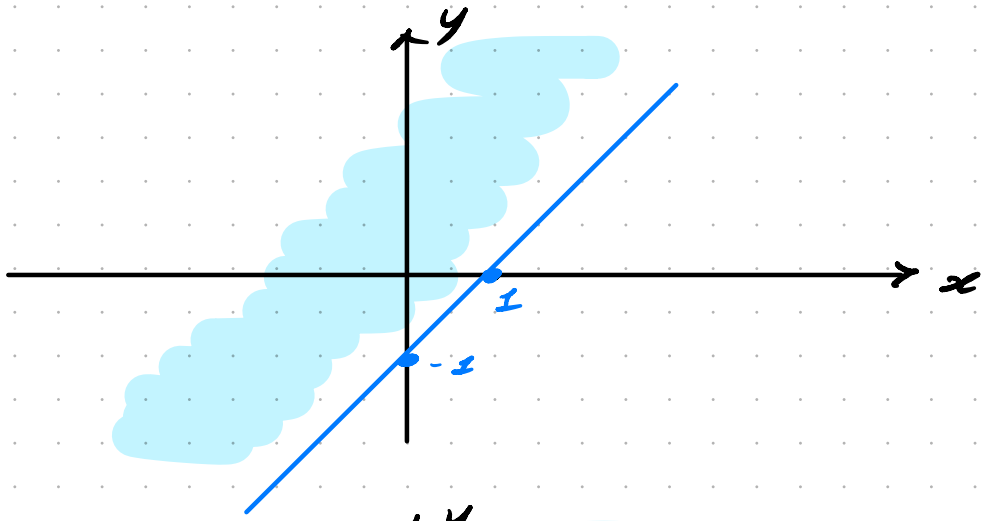


ALTRE RISPOSTE:

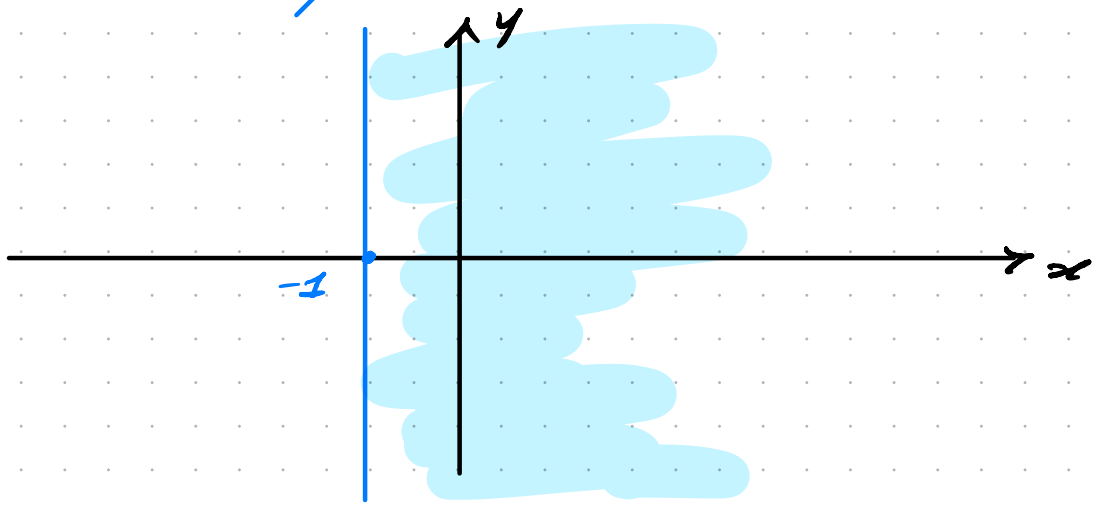
$y < -1 \Rightarrow$



$y > x - 1 \Rightarrow$

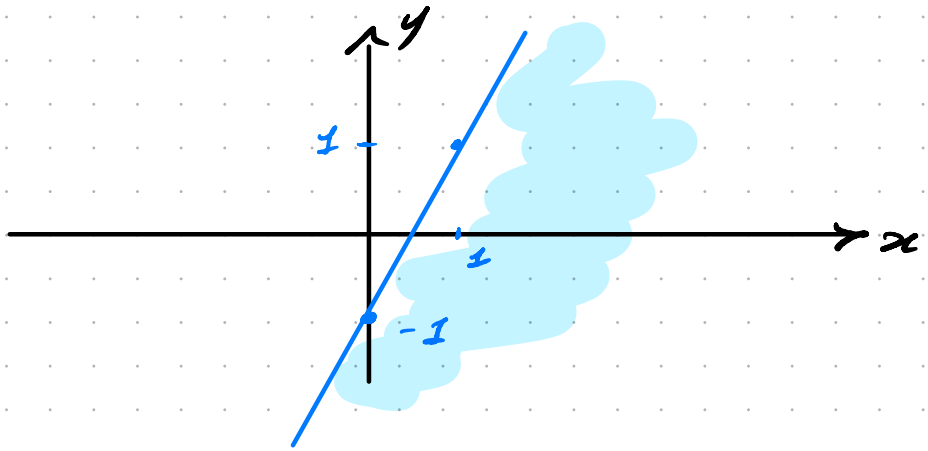


$x > -1 \Rightarrow$



ex

- $y < 2x - 1$



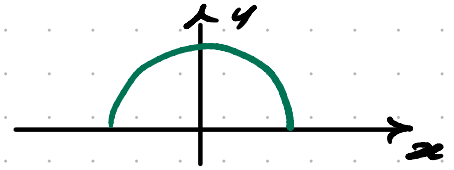
CIRCONFERENZA

$C(x_0, y_0) \rightarrow (x-x_0)^2 + (y-y_0)^2 = r^2$

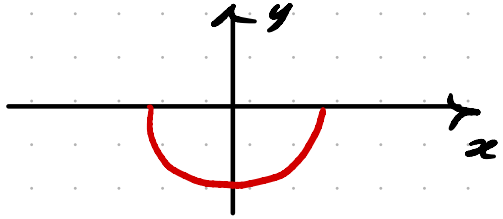
$x_0 = y_0 = 0 \rightarrow x^2 + y^2 = r^2$

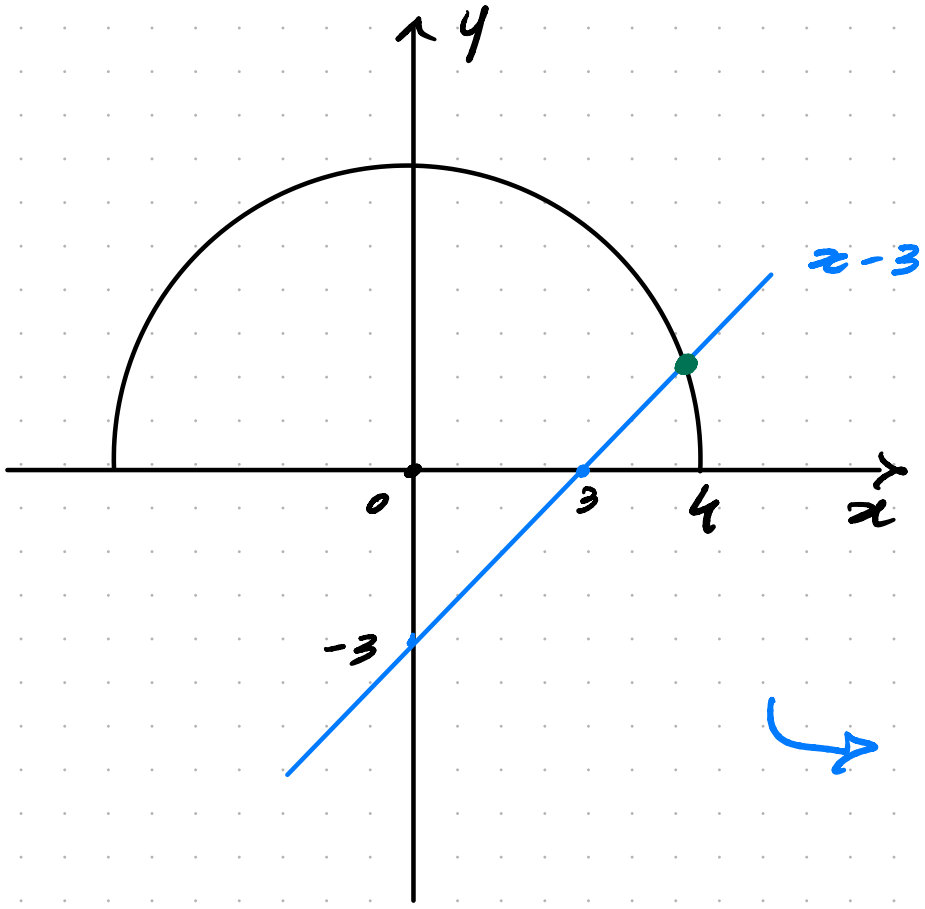
$y^2 = r^2 - x^2 \Rightarrow y = \pm\sqrt{r^2 - x^2}$

$y = \sqrt{r^2 - x^2} \geq 0 \rightarrow$



$y = -\sqrt{r^2 - x^2} < 0 \rightarrow$





$\sqrt{16 - x^2} = x - 3$

ALTRE RISPOSTE

$\sqrt{x^2+16}$ → IPERBOLE EQUILATERA

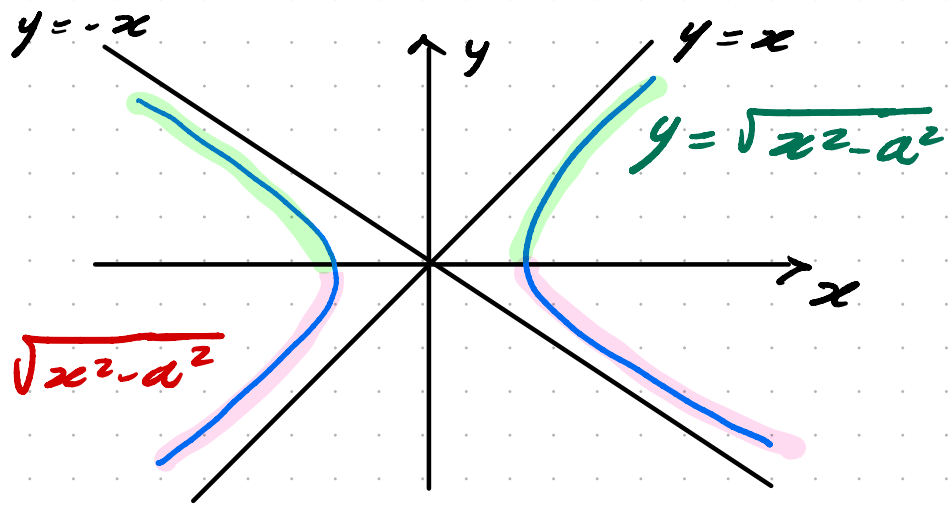
$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$y = \pm \frac{b}{a} x$

↳ asintoti

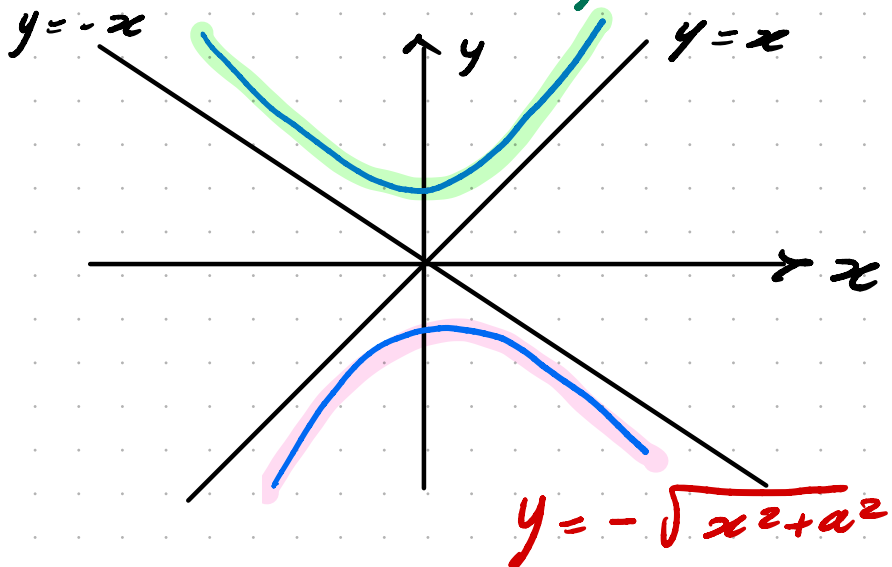
→ EQUILATERA ⇒ $a = b$ ⇒ $x^2 - y^2 = a^2$, $y = \pm x$

⇒ $y^2 = x^2 - a^2$ ⇒ $y = \pm \sqrt{x^2 - a^2}$



$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ → $y^2 - x^2 = a^2$

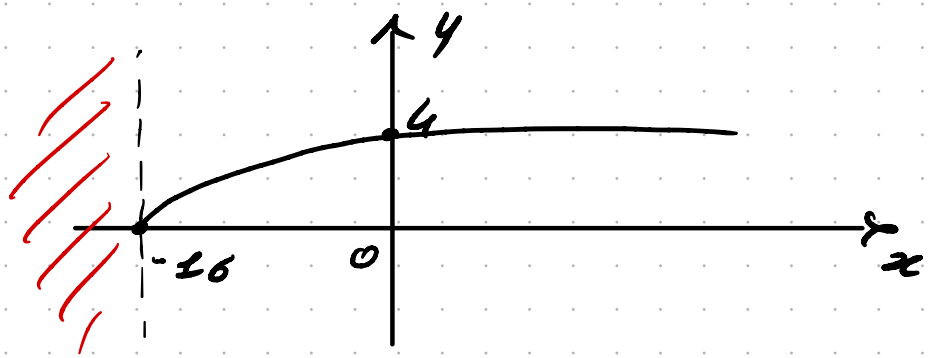
⇒ $y = \pm \sqrt{x^2 + a^2}$



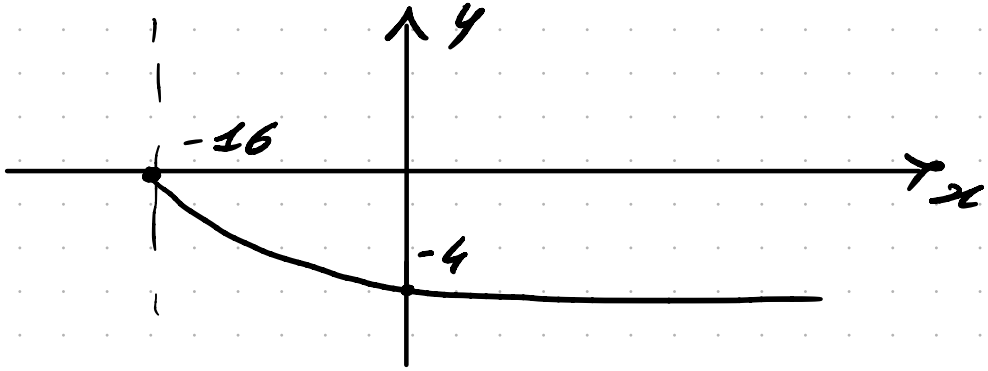
$\sqrt{x+16}$ → ОДЗ $x+16 \geq 0 \Rightarrow x \geq -16$

$y = +\sqrt{x+16}$

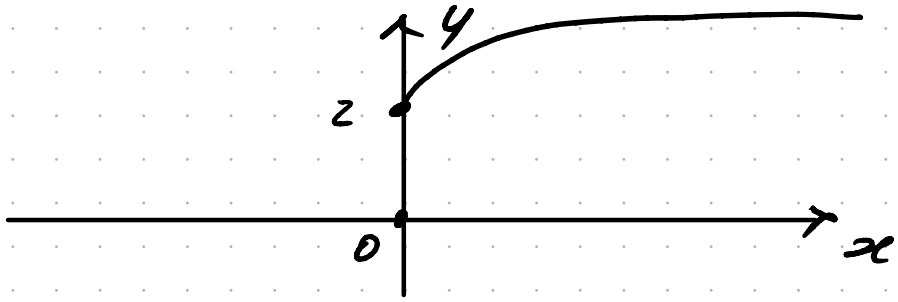
$x = 0 \rightarrow y = \sqrt{16} = 4$

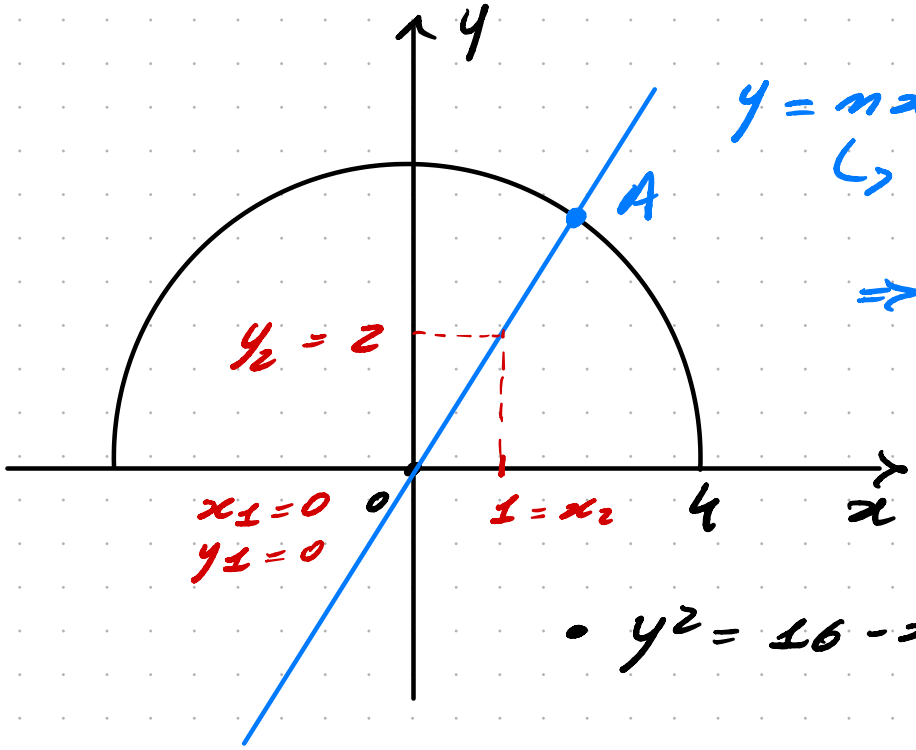


$y = -\sqrt{x+16}$



$y = \sqrt{x} + 2$





$y = mx$, $q = 0$

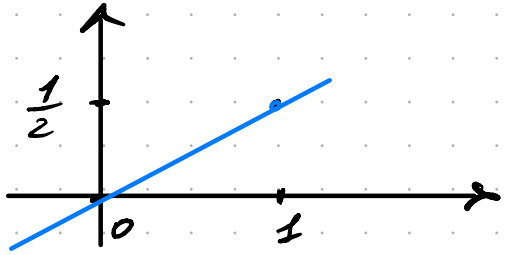
↳ $m > 0$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2}{1} = 2$

$\Rightarrow y = 2x$

- $y^2 = 16 - x^2 \rightarrow x^2 + y^2 = 16$ → NO!
  ↳ CIRCONFERENZA
- $y = \sqrt{16 - x^2}$ ⇒ ✓
- $y = \sqrt{x^2 - 16}$ → IPERBOLE PARTE POSITIVA → NO!

↳ $\begin{cases} y = 2x \\ y = \sqrt{16 - x^2} \end{cases}$

↳ $\begin{cases} y = \frac{1}{2}x \rightarrow \\ y = \sqrt{x^2 - 16} \end{cases}$



$$\begin{cases} x > 0 \\ 5 - x^2 < 0 \end{cases}$$

A → $\sqrt{5 - x^2} < x$ → $\begin{cases} 5 - x^2 \geq 0 \\ x > 0 \\ \sqrt{5 - x^2} < x \end{cases}$ → $\begin{cases} 5 - x^2 \geq 0 \\ x > 0 \\ 5 - x^2 < x^2 \end{cases}$ → $\begin{cases} 5 - x^2 \geq 0 \\ x > 0 \\ 5 - 2x^2 < 0 \end{cases}$ ≠

B → $\sqrt{15 + x^2} < 2x$ → $\begin{cases} x > 0 \\ 15 + x^2 < 4x^2 \end{cases}$ → $\begin{cases} x > 0 \\ 15 - 3x^2 < 0 \end{cases}$

→ $\begin{cases} x > 0 \\ 5 - x^2 < 0 \end{cases}$ → ✓

C → $\sqrt{5 + x^2} < 2x$ → $\begin{cases} x > 0 \\ \sqrt{5 + x^2} < 2x \end{cases}$ → $\begin{cases} x > 0 \\ 5 + x^2 < 4x^2 \end{cases}$

→ $\begin{cases} x > 0 \\ 5 - 3x^2 < 0 \end{cases}$ → ≠

D → $\sqrt{5 + x^2} > 2x$ → $\begin{cases} 2x \leq 0 \\ 5 + x^2 \geq 0 \\ \uparrow \text{sempre} \end{cases}$ ∪ $\begin{cases} 2x > 0 \\ 5 + x^2 > 4x^2 \end{cases}$

→ $\begin{cases} x \leq 0 \end{cases}$ ∪ $\begin{cases} x > 0 \\ 5 - 3x^2 > 0 \end{cases}$ → ≠

es

$\sqrt[n]{f(x)} = g(x)$

- $\sqrt[3]{x^2+11x+27} = x+3$ → RADICE DISPARI

$\Rightarrow x^2+11x+27 = (x+3)^3$ ↳ n disp → $a=b \Leftrightarrow a^n = b^n \quad \forall a,b \in \mathbb{R}$

$x^2+11x+27 = x^3+9x^2+27x+27$

$x^3+8x^2+16x = 0 \Rightarrow x(\underbrace{x^2+8x+16}_{(x+4)^2}) = 0$ → $x_1 = 0$; $x_2 = -4$

- $\sqrt{2-x+(x-1)^2} + 2x - 1 = 0$ → RADICE PARI

$\sqrt{2-x+x^2-2x+1} = 1-2x$

$\sqrt{x^2-3x+3} = 1-2x$

↳ n pari → $a=b \Leftrightarrow a^n = b^n \quad \forall a,b \in \mathbb{R}$ e concordi

$\Rightarrow \begin{cases} f(x) = [g(x)]^n \\ g(x) \geq 0 \end{cases}$

$\begin{cases} x^2-3x+3 = (1-2x)^2 \\ 1-2x \geq 0 \end{cases}$

$\begin{cases} x^2-3x+3 = 1-4x+4x^2 \\ 1-2x \geq 0 \end{cases} \Rightarrow \begin{cases} 3x^2-x-2 = 0 \\ x \leq \frac{1}{2} \end{cases}$

$\Rightarrow \begin{cases} x_{1/2} = \frac{+1 \pm \sqrt{25}}{6} \\ x \leq \frac{1}{2} \end{cases} \Rightarrow$

$x_1 = \frac{1+5}{6} = 1 > \frac{1}{2}$ NON ACCETTABILE

$x_2 = \frac{1-5}{6} = -\frac{2}{3} \leq \frac{1}{2}$ OK!!!