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Matematica creativa e packaging

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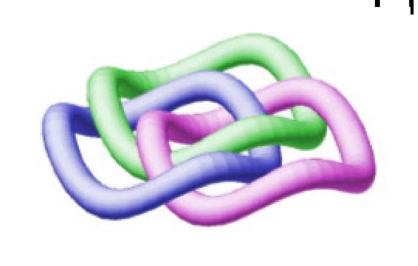

an **informa** business

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POLIGONI E TASSELLAZIONI DEL PIANO

an informa business

I poligoni regolari

Ottimizziamo:

Dati un triangolo equilatero, un quadrato, un esagono regolare con la stessa area *A*, quale di essi ha perimetro minore?

$$\ell_3 = \frac{2}{\sqrt[4]{3}}\sqrt{A} \cong 1.5197\sqrt{A} \qquad \rightarrow P_3 \cong 4.559\sqrt{A}$$

$$\ell_4 = \sqrt{A} \qquad \rightarrow P_4 = 4\sqrt{A}$$

$$\ell_6 = \frac{\sqrt{2}\sqrt[4]{3}}{3}\sqrt{A} \cong 0.6204\sqrt{A} \rightarrow P_6 \cong 3.7224\sqrt{A}$$

$$A_3 = A_4 = A_6 \rightarrow P_3 < P_4 < P_6$$

I poligoni regolari

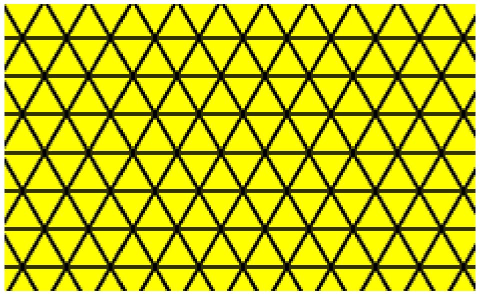
Ottimizziamo:

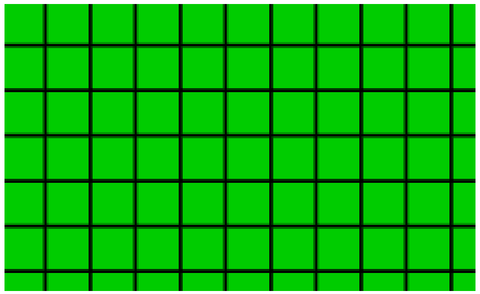
Dati un triangolo equilatero, un quadrato, un esagono regolare con lo stesso perimetro è l'esagono ad avere area maggiore

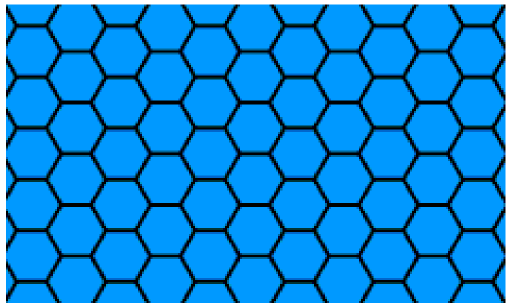
$$P_3 = P_4 = P_6 \rightarrow A_3 < A_4 < A_6$$

Tassellazione regolare

E’ possibile ricoprire il piano con poligoni regolari dello stesso tipo se e solo se sono triangoli equilateri, quadrati, esagoni regolari.



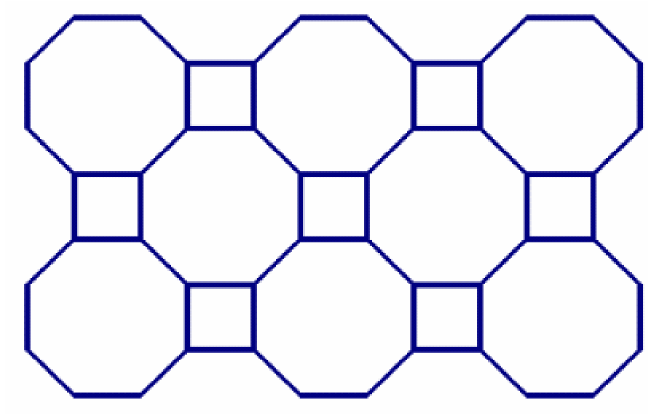


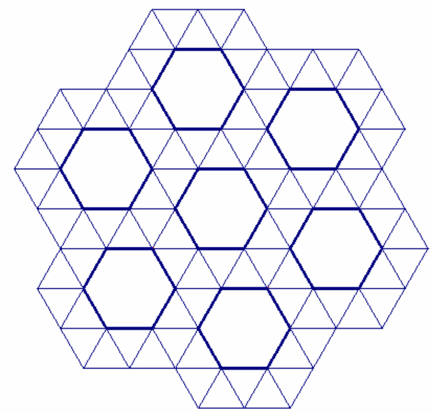


Tassellazione semi-regolare

Ottenuta con poligoni regolari (diversi fra loro).

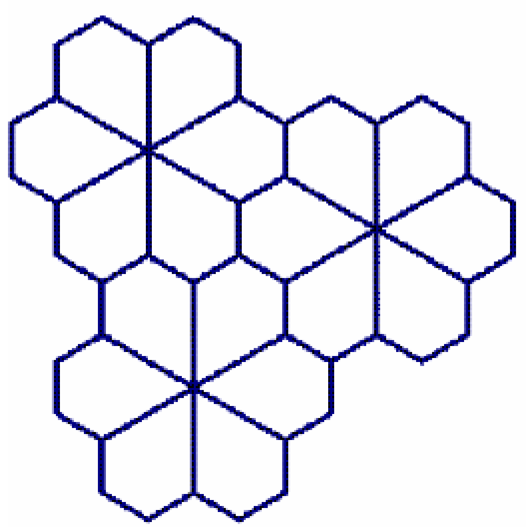
In tutti i vertici la disposizione dei poligoni è la stessa

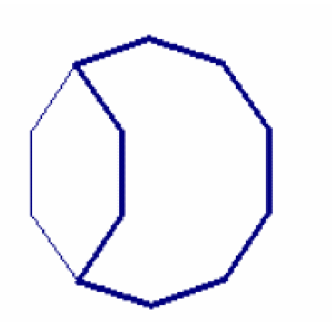


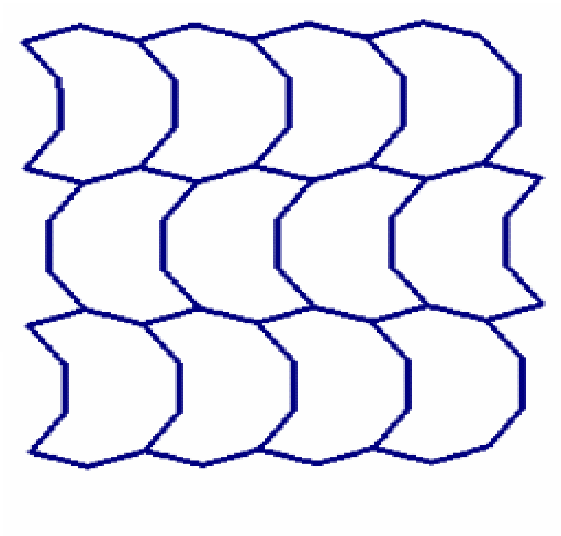


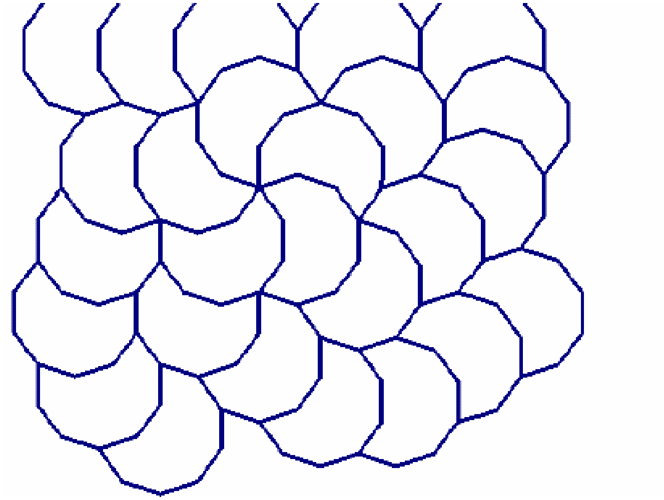
Tassellazioni quasi-regolari

Forme non regolari **tutte uguali** tra loro





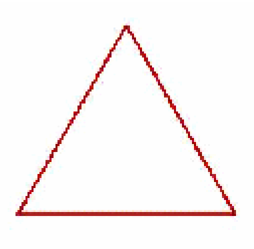


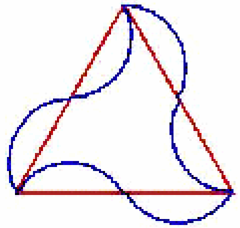


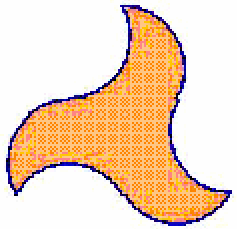
Tassellazioni quasi-regolari

Pajarita Nazarì

Ha la stessa area del triangolo equilatero da cui si ottiene.

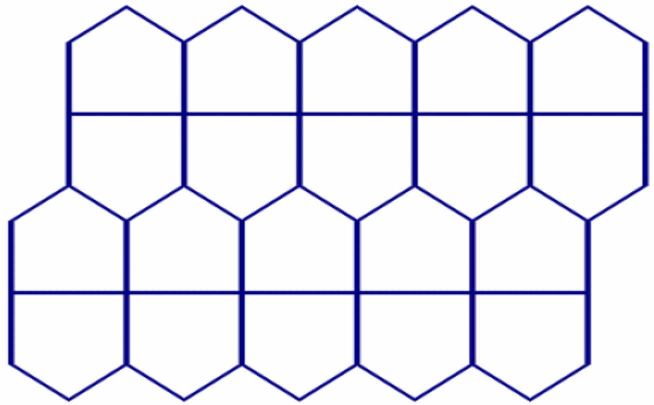


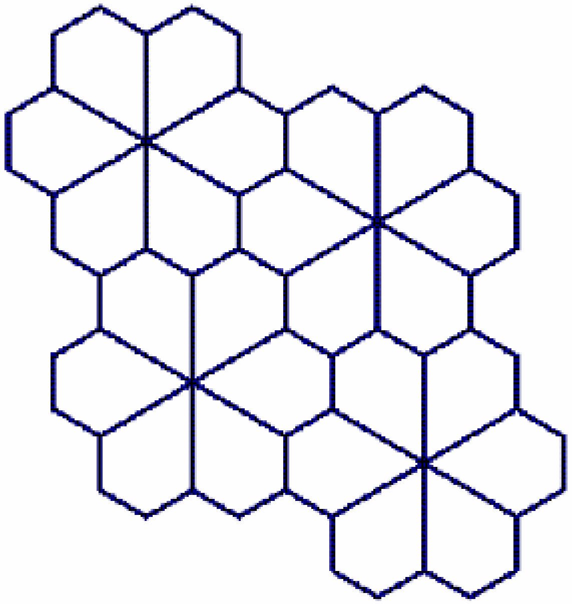


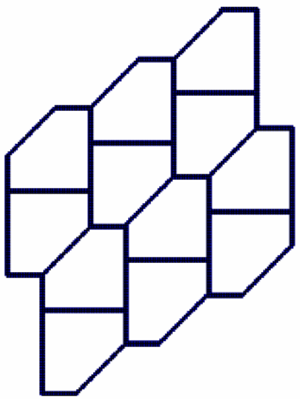


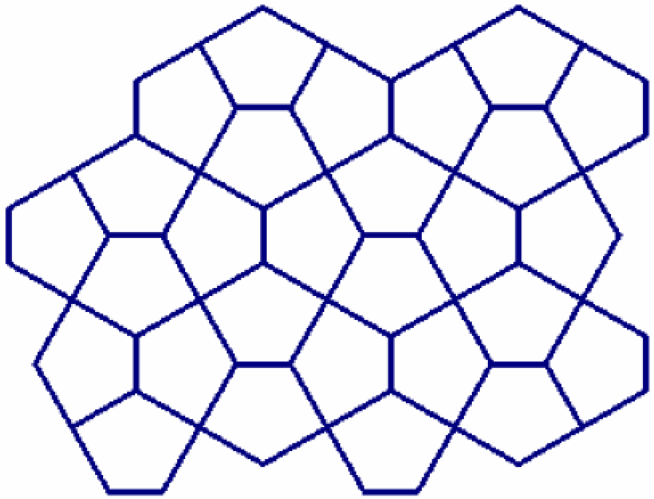
Tassellazione con pentagoni

*La tassellazione del piano con **pentagoni non regolari** è **possibile**.*









DAI POLIGONI REGOLARI AI POLIGONI DI REULEAUX

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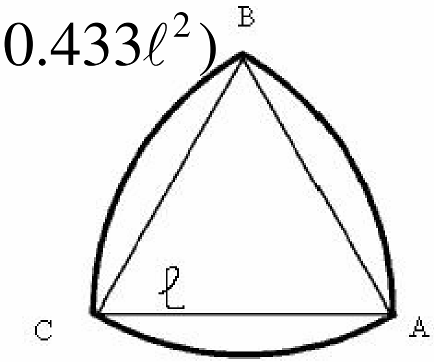
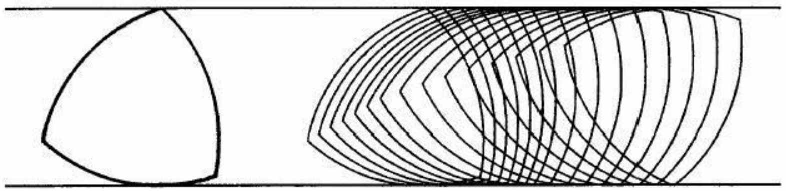
Triangolo di Reuleaux

***Curva chiusa**, **figura di larghezza costante**:*

- *tre archi di circonferenza centrati nei vertici di un triangolo equilatero di lato* ℓ
- *rotola tra due rette parallele come una circonferenza*

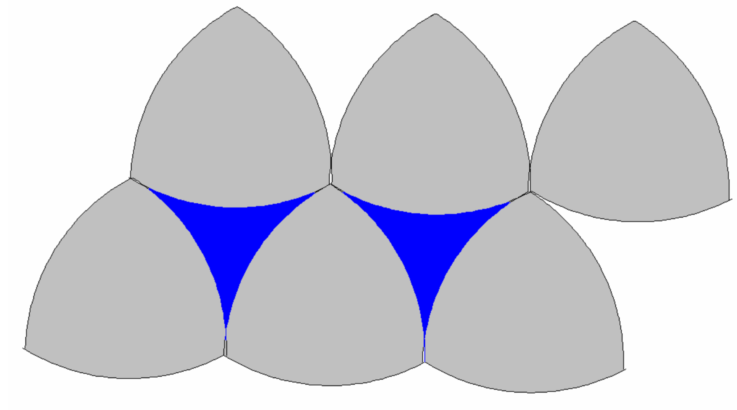
- *lunghezza* $\pi\ell \quad (P_3 = 3\ell)$
- *area* $\dfrac{(\pi - \sqrt{3})}{2}\ell^2 \cong 0.7048\ell^2 \quad (A_3 = \dfrac{\sqrt{3}}{4}\ell^2 \cong 0.433\ell^2)$





Tassellazione quasi-regolare

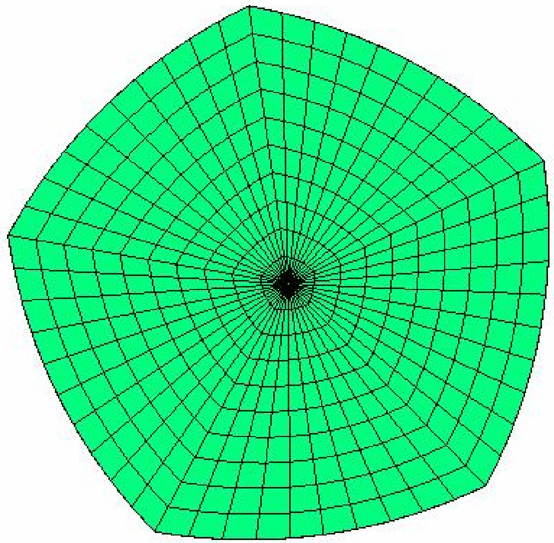
Le proprietà geometriche del ***triangolo di Reuleaux*** *permettono una* ***tassellazione quasi-regolare*** *del piano*



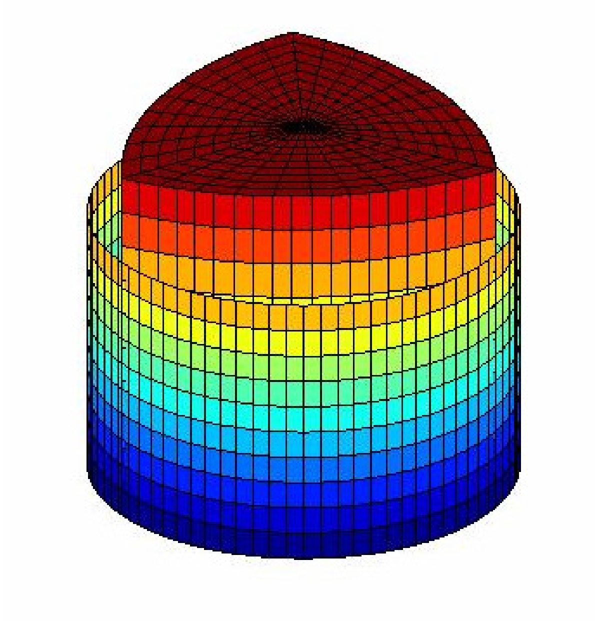
Poligoni di Reuleaux

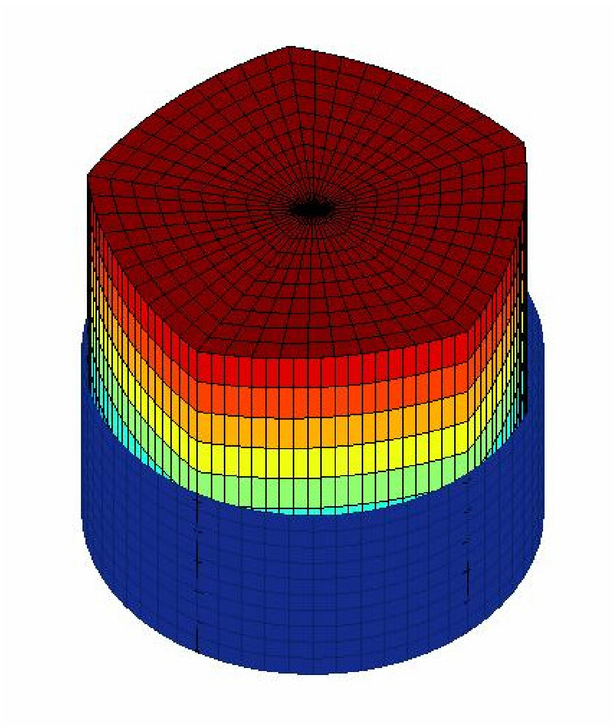
***Curve chiuse**, **figure di larghezza costante**:*

- *archi di circonferenza centrati nei vertici di un poligono regolare di 2n+1 lati*
- *rotolano tra due rette parallele come una circonferenza*



Scatole di Reuleaux



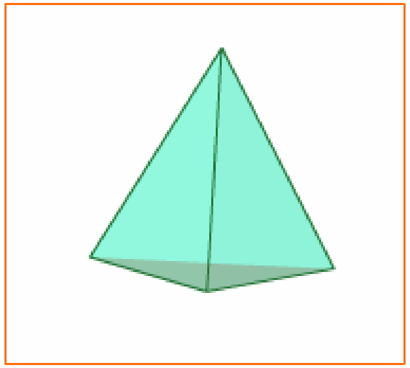


Un poligono di Reuleaux è inscritto nella stessa circonferenza del poligono regolare

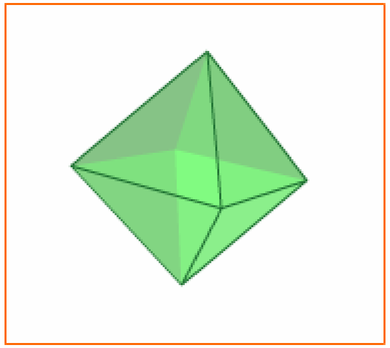
POLIEDRI

I solidi platonici

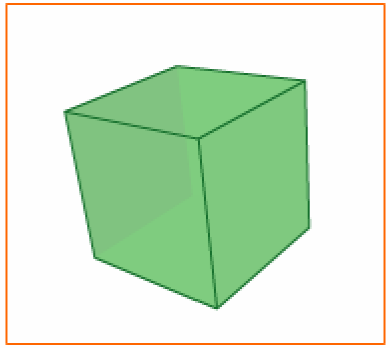
Tetraedro



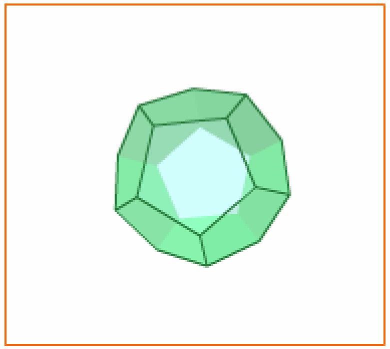
Ottaedro



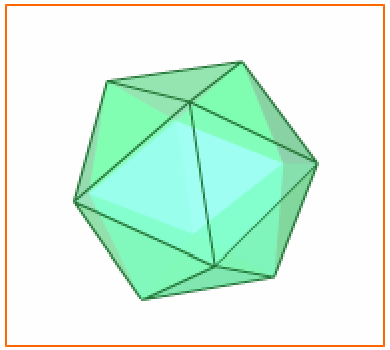
Cubo o esaedro



Dodecaedro



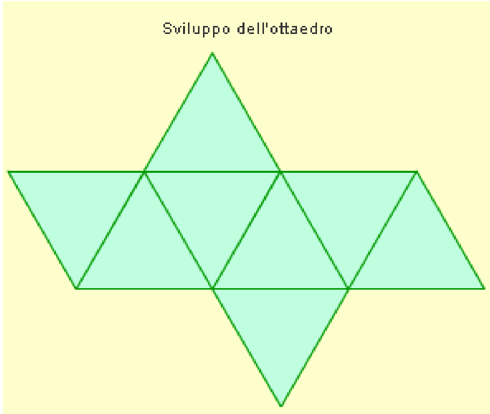
Icosaedro

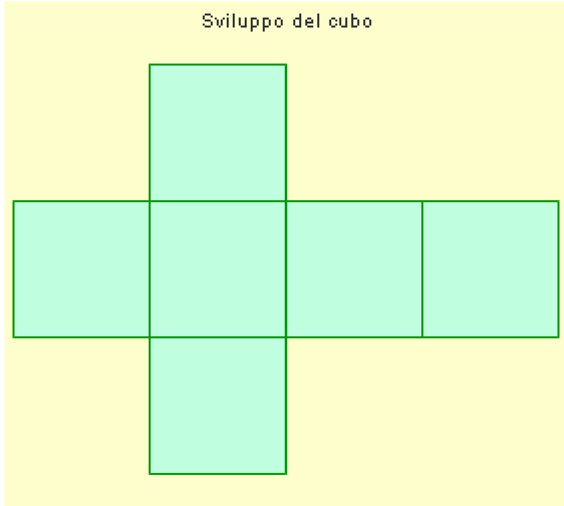


Solidi convessi *limitati da* ***poligoni regolari*** *tutti* ***uguali*** *tra loro.*

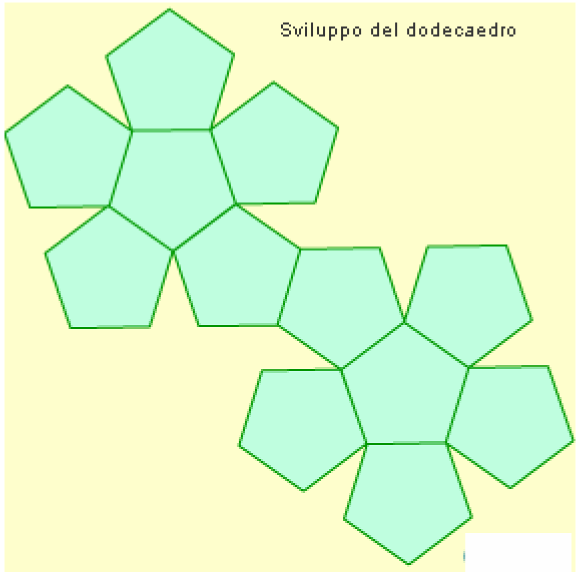
Sono solo ***cinque****.*

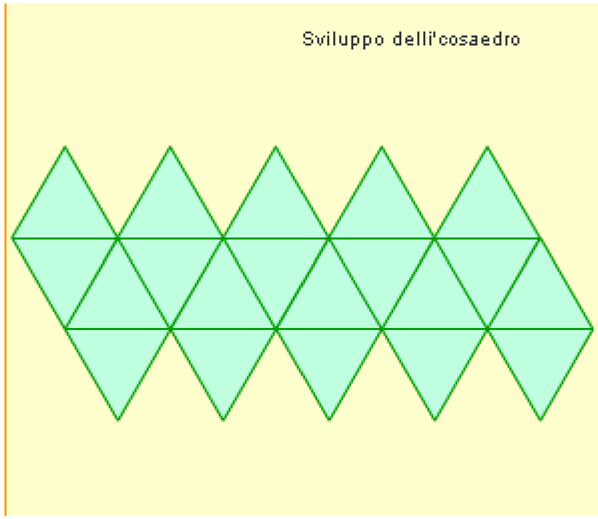
I solidi platonici



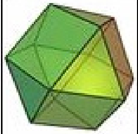
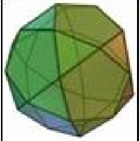
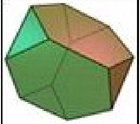
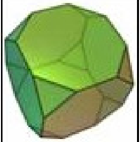
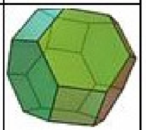
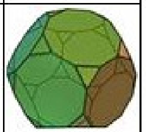
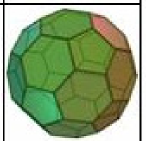


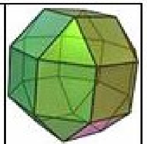
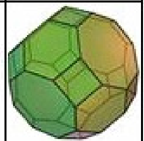
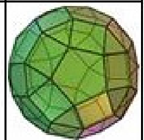
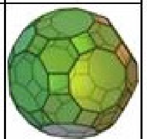
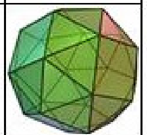
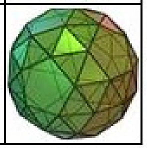
Le facce dei poliedri sono triangoli equilateri, quadrati, pentagoni.

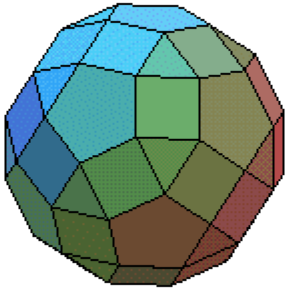




I solidi di Archimede

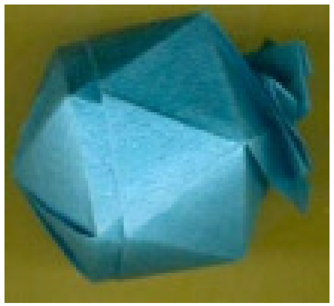
cubottaedro	
icosidodecaedro	
tetraedro troncato	
cubo troncato (o esaedro troncato)	
ottaedro troncato	
dodecaedro troncato	
icosaedro troncato (o pallone da calcio)	

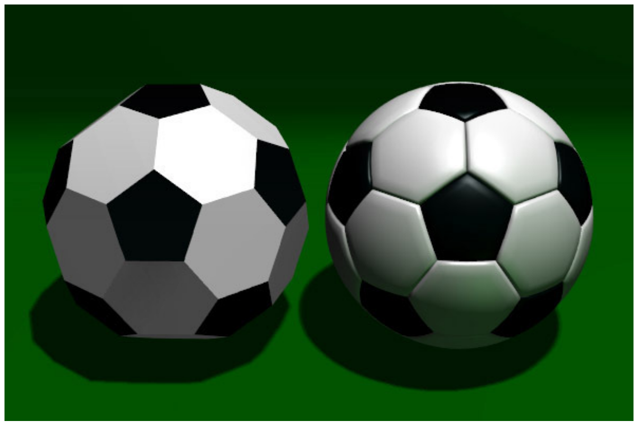
rombicubottaedro	
cubottaedro troncato	
rombicosidodecaedro	
icosidodecaedro troncato)	
cubo camuso	
dodecaedro camuso	



*Solidi convessi delimitati da **due o tre tipi di poligoni regolari.** I più semplici si ottengono troncando i vertici dei solidi platonici.*

Solidi platonici e archimedei



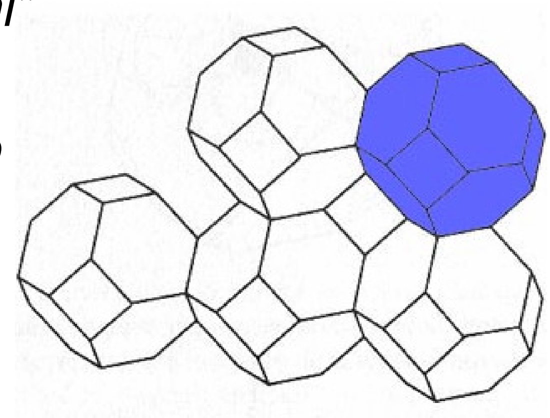


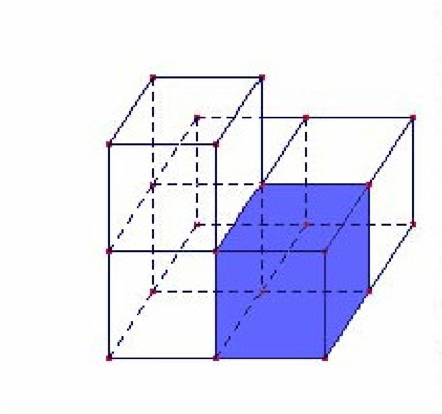
Tassellazione dello spazio

*Insieme di poliedri adiacenti che riempiono **tutto** lo spazio, senza lasciare "buchi"*

*Tra i solidi platonici è solo il **cubo** .*

*Tra i solidi archimedei è solo **l'ottaedro tronco.***

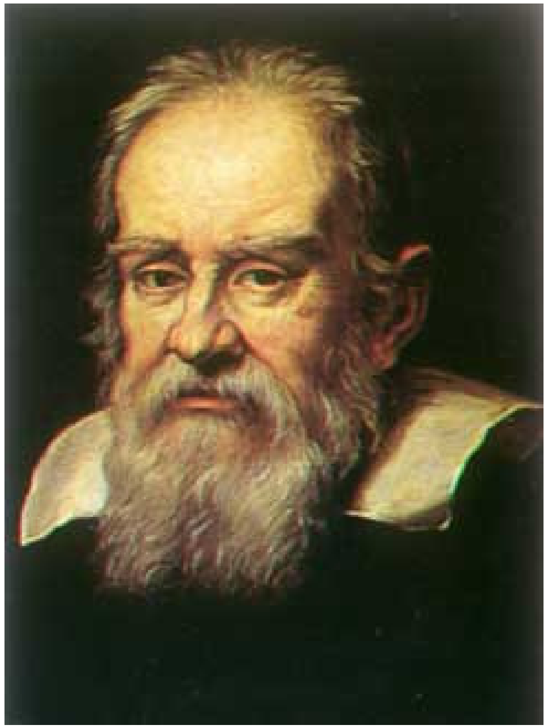


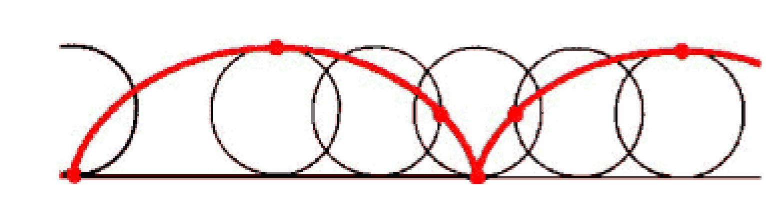


CURVE E SUPERFICI

Cicloide

Curva aperta *tracciata da un punto appartenente ad una* ***circonferenza*** *che* ***rotola lungo una retta***

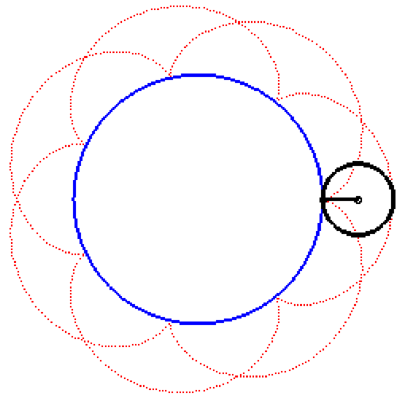




Epicicloide

Curva tracciata da un punto appartenente ad una ***circonferenza*** *di raggio r che* ***rotola esternamente ad un'altra circonferenza*** *di raggio a (a>r)*

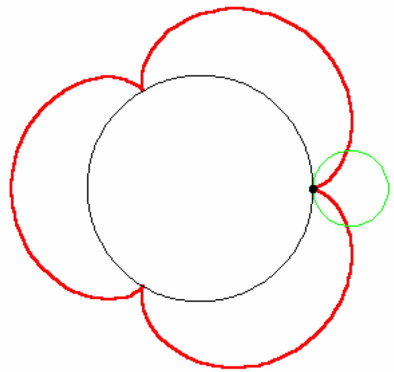
$$L = \frac{8(q+1)}{q^2} a$$



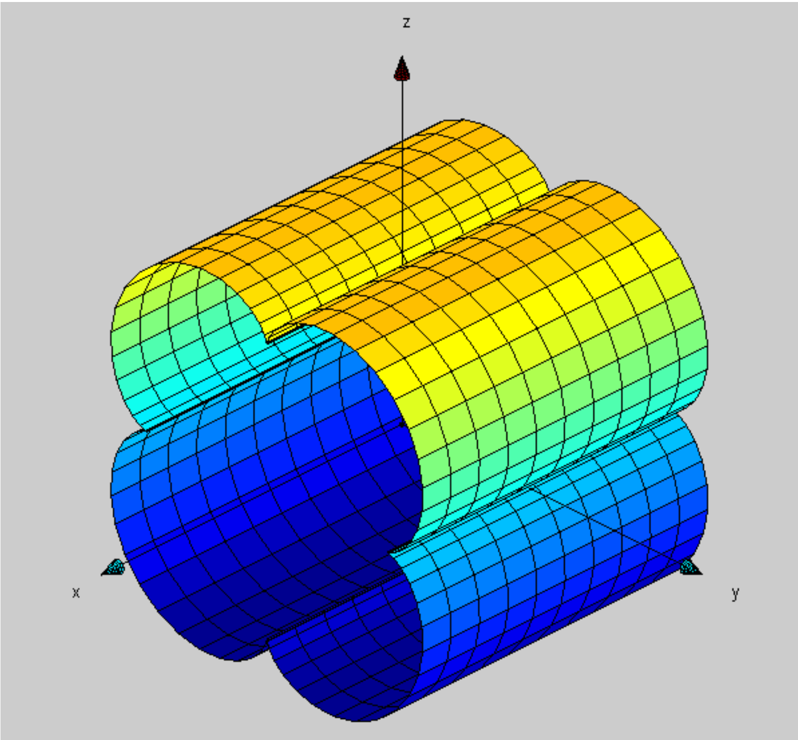
Chiusa *se q =a/r è razionale*

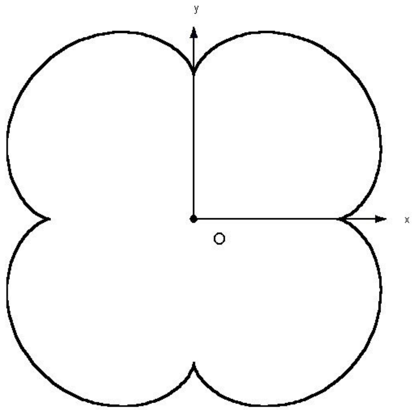
Aperta *se q =a/r è irrazionale*

$$q = 3 \rightarrow L_{tot} = \frac{32}{3} a \quad (> 2\pi a)$$



Epicicloide



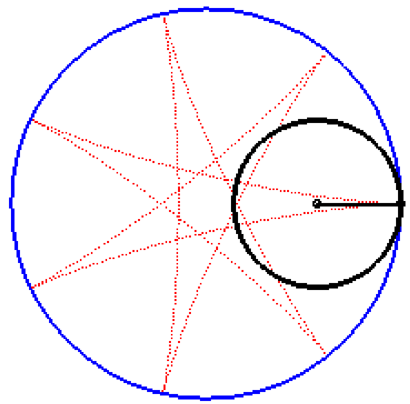


$$q = 4 \rightarrow L_{tot} = 10a \quad (> 2\pi a)$$

Ipocicloide

*Curva tracciata da un punto appartenente ad una **circonferenza** di raggio r che **rotola internamente** ad **un'altra circonferenza** di raggio a (a>r)*

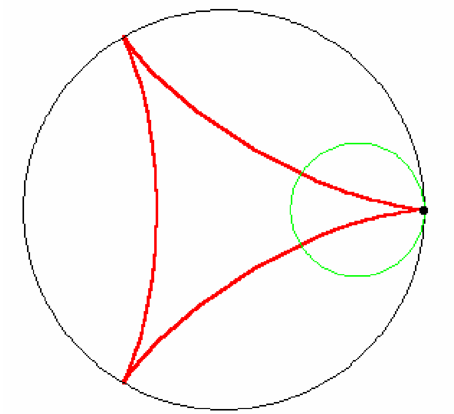
$$L = \frac{8(q-1)}{q^2} a$$



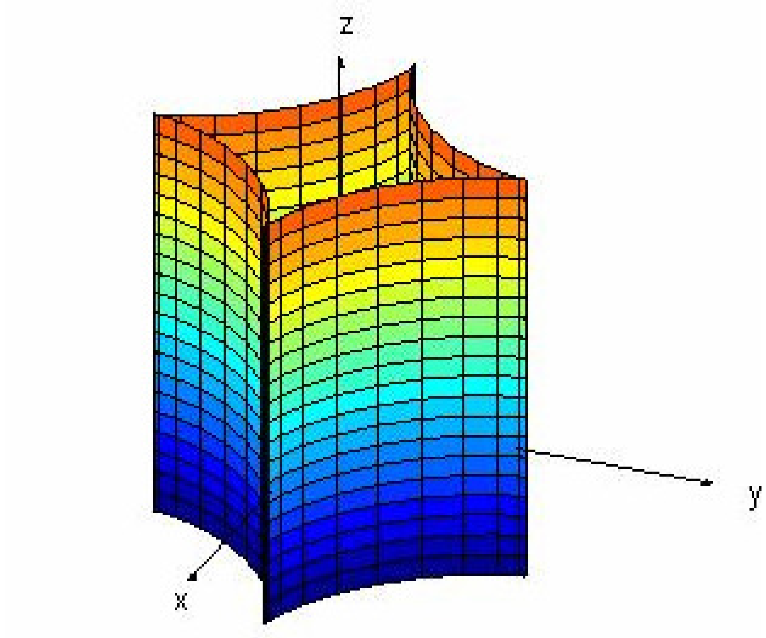
***Chiusa** se q =a/r è razionale*

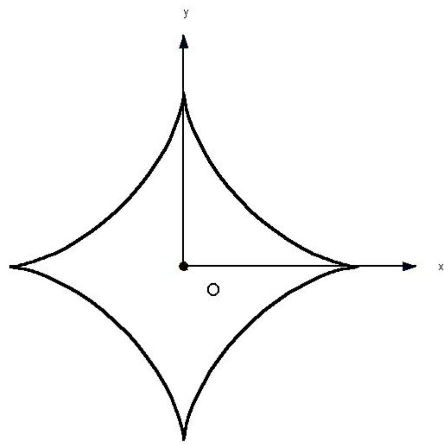
***Aperta** se q =a/r è irrazionale*

$$q = 3 \rightarrow L_{tot} = \frac{16}{3} a \ (< 2\pi a)$$



Ipocicloide





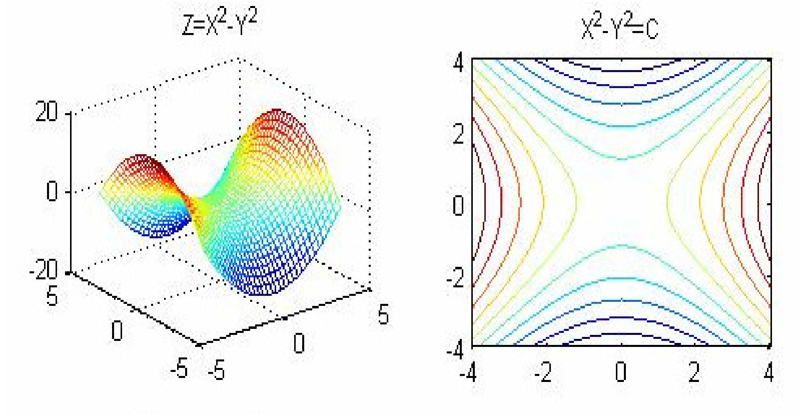
$$q = 4 \rightarrow L = 6a \quad (< 2\pi a)$$

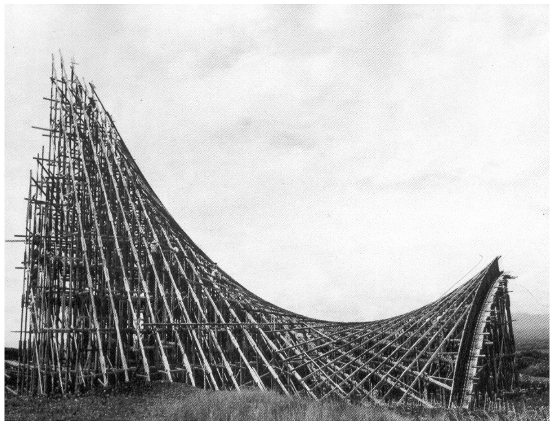
Paraboloide iperbolico

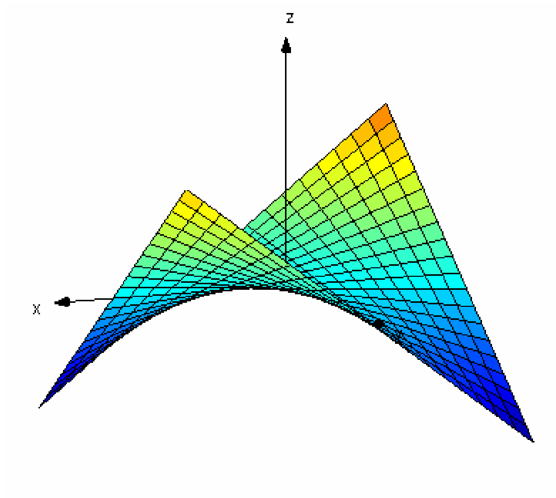
*E' una superficie a **doppia curvatura**. Ha la forma di una **sella**.*

*Le sezioni con piani orizzontali sono **iperboli**, le sezioni con piani verticali sono **parabole**.*

*E' una superficie **rigata**: per ogni suo punto passa almeno una retta.*





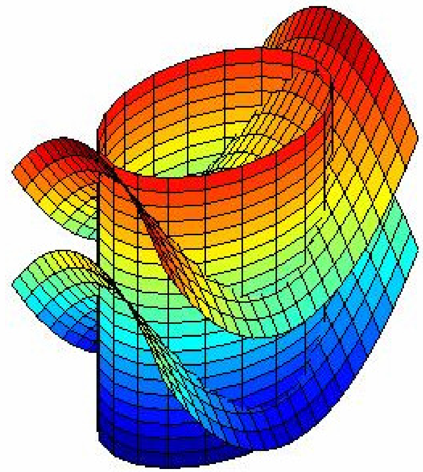


Paraboloide iperbolico

Le patatine fritte assumono una configurazione a sella: la scatola rispetta la loro forma

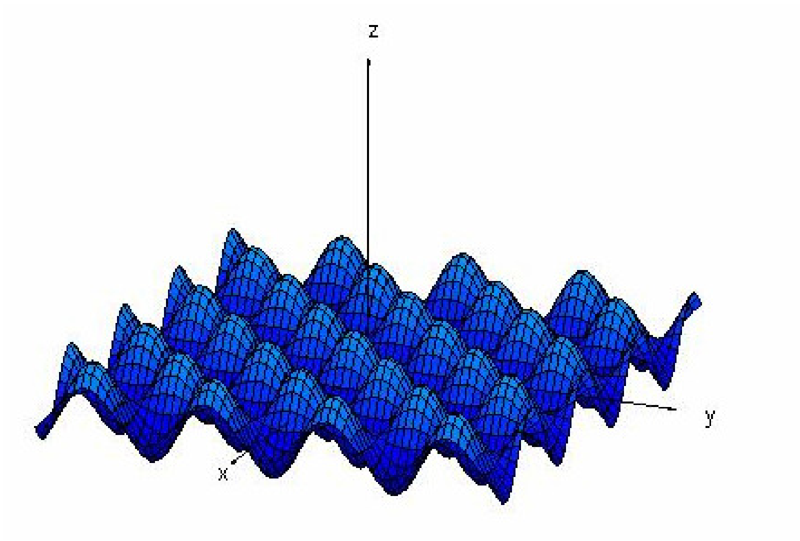
Paraboloide iperbolico

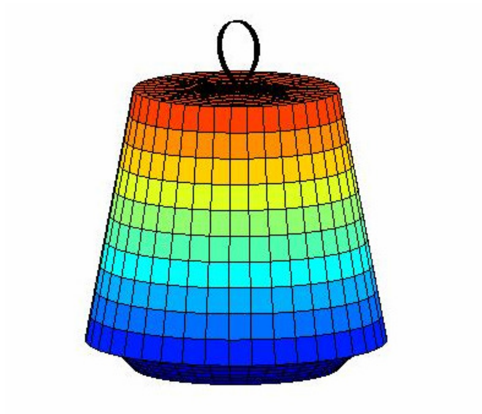
La scatola verde si può ottenere intersecando un ***cilindro a sezione ellittica*** *con due* ***paraboloidi iperbolici***



Modello matematico

Molte confezioni si possono interpretare in termini matematici.





Istituto Internazionale di Ricerca
Know how to achieve

Le trasformazioni lineari

La duplicazione del cubo

*I cubi sovrapposti hanno ciascuno **volume metà** del precedente*

*La "torre" è generata trasformando il cubo alla base di spigolo L_1 mediante **riduzione e traslazione**.*

*Si può valutare la lunghezza dello spigolo di ogni cubo tenendo conto del **fattore di riduzione**.*

$$L_2 = \frac{L_1}{\sqrt[3]{2}},\ L_3 = \frac{L_2}{\sqrt[3]{2}} = \frac{L_1}{(\sqrt[3]{2})^2},\dots$$

$$L_n = \frac{L_{n-1}}{\sqrt[3]{2}} = \frac{L_1}{(\sqrt[3]{2})^n}$$